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Market Structure and Monopoly Profits: A Dynamic Theory

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In any discussion of antitrust policy, the relationship of market share to monopoly power is an issue of central importance. In practice, the basis upon which a particular share is considered "substantially to lessen competition or to tend to create a monopoly" is not at all clear. In view of the virtual absence of economic theory relevant to this question, one may well be hesitant in criticizing the use of more or less arbitrary rules of thumb to decide whether a particular market share is dangerous to competition. It may, however, be of interest to inquire whether existing methods of analysis can widen our understanding of the problem and provide better criteria for antitrust policy. This paper is an attempt to open such an inquiry.

The analysis departs from conventional oligopoly theory in three important respects. First, constant returns to scale, a hypothesis consistent with a large body of empirical evidence, is assumed. The conventional U-shaped cost curve appears to be inconsistent with the observed wide divergence of firm sizes in an industry and Gibrat's law [10] that growth rates are independent of firm size. Second, the theory is dynamic which eliminates the ambiguity between the short and the long run. This is accomplished by assuming increasing costs associated with rapid adjustment in capacity. The final departure is the assumption of rational expectations as to current and future output of rival firms. This assumption may seem unreasonable, but I argue it is preferable to any other hypothesis. Any alternative expectation scheme implies businessmen consistently will make easily corrected forecast errors. These errors will be costly unless the scheme is approximately correct or in other words, approximately rational. Thus, the rational expectation solution will provide a good approximation for any expectation scheme consistent with businessmen not making costly and easily corrected errors.[1]

[1] Institutional arrangements will assist in this forecasting. For example, most industries have trade journals which keep firms informed as to growth plans of rivals

The first section of the paper presents the model and defines what is meant by a solution path for market structure. Section 2 develops the short run and the long run stationary solutions. Simple relationships are presented for the stationary market shares as a function of economic profits per unit output. The optimization problem facing a single firm is considered in the next section. These results are then used to prove noncooperative equilibrium exists to this n firm infinite period gaming problem and to prove any equilibrium path necessarily converges to the long run stationary solution. The final section contains concluding remarks.

1. The Model

An industry with n firms each experiencing constant returns to scale, but increasing costs associated with rapid adjustments in capacity is considered. The output of firm i in period t, denoted by q_{it}, is constrained by its capacity k_{it}

$$0 \leqslant q_{it} \leqslant k_{it}, \qquad t = 0, 1, 2,\dots . \tag{1}$$

Short run marginal costs are a constant, c_i, in this range. The firm's capacity in the subsequent period depends upon the current investment x_{it} and present capacity as follows:

$$k_{i,t+1} = k_{it}\, h(x_{it}/k_{it}), \qquad t = 0, 1, 2,\dots . \tag{1.2}$$

The function $h(\cdot)$ is assumed to be continuously differentiable, increasing, and strictly concave. It is further assumed that $h(\delta) = 1$ for some $0 < \delta < 1$ and $h(0) > 0$. These assumptions imply that δk_{it} is the investment rate which will just maintain the stock k_{it} and that some capacity remains in period $t + 1$ even if there is no investment in period t. An alternative formulation of the investment relationship is obtained by solving (1.2) for x_{it}

$$x_{it} = k_{it}\, h^{-1}(k_{i,t+1}/k_{it}) = k_{it}\, g(k_{i,t+1}/k_{it}), \tag{1.3}$$

a representation which also will be used. The function g is strictly convex, increasing, and continuously differentiable given the assumed properties of h.

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and provide forecasts of future outputs and prices. In a world with uncertainty, rational expectations do not imply perfect foresight, but rather the subjective probability distributions of outcomes coincides with that distribution predicted by the model. See [8] for an application of this concept.

Given an initial stock k_{i0}, relationship (1.2) cannot be solved for k_{it} as a *linear* function of k_{i0}, x_{i0},..., $x_{i,t-1}$, as with conventional depreciation hypotheses, so perhaps it warrants some explanation. One plausible possibility is that the true relationship between physical capacity is, in fact, nonlinear, so that a given quantity of say, machines, make a better plant (yield more productive services) the longer the period over which they are assembled. Alternatively, one may assume that the relation between capacity and physical investment is indeed linear, and regard x_{it} as the dollar value of investment as suggested by (1.3). Then, if investment costs per unit of capacity are a strictly convex function of physical investment per unit of cpapacity, (1.2) is implied.[2]

The demand in period t along with industry output determine the market price in period t

$$p_t = D(Q_t) \qquad \text{with} \quad Q_t = \sum_{i=1}^{n} q_{it}, \tag{1.4}$$

where $D'(\cdot) < 0$, $D(0) < \infty$, and the industry marginal revenue has negative slope. It is further assumed there is a $\bar{k}$ such that $\bar{k} \geqslant k_{i0}$ for all i and $D(\bar{k}) \leqslant \min c_i$. Clearly, no firm will ever choose a path such that $k_{it} \geqslant \bar{k}$ for any t, which bounds outputs and capacity to the internal $[0, \bar{k}]$.

The objective of each of the n firms is to maximize its expected present value given the expected output sequence for rivals e_i

$$\pi_i = \sum_{t=0}^{\infty} \beta^t \{[D(e_{it} + q_{it}) - c_i]\, q_{it} - x_{it}\}, \qquad i = 1, 2, \ldots, n, \tag{1.5}$$

where $\beta = 1/(1 + r)$ is the discount factor and r the interest rate. In the next section, it is shown that the unique maximizing sequence $\mathbf{q}_i = \{q_{it}\}$ will be a function of firm i expectations $\mathbf{e}_i$ as to the behavior of rivals

$$\mathbf{q}_i = T_i(\mathbf{e}_i). \tag{1.6}$$

An equilibrium solution to this problem is an n-tuple of sequences such that

$$\mathbf{q}_i = T_i\left(\sum_{j \neq i} \mathbf{q}_j\right) = T_i(\mathbf{Q}_i) \qquad \text{for all } i.$$

In other words, a solution is characterized by expectations being rational in the sense that what people expect to happen occurs.[3]

[2] This structure was used by [8]. Other assumptions which would imply increasing costs associated with rapid adjustment in capacity and be consistent with constant returns to scale, such as those of [6] and [7], would not have altered the conclusions.

[3] Muth [9] used the term rational expectation for models with uncertainty, but the concept seems appropriate for deterministic dynamic processes as well. Brock [3] has called this the equilibrium forecasts, namely a set of forecasts which imply behavior that results in outcomes consistent with the forecasts.

2. Short Run and Long Run Solutions

Current industry structure will determine current output and price. In equilibrium, the short run marginal revenue of firm i will be

$$D(Q_t) + D'(Q_t)q_{it}\,. \tag{2.1}$$

Equating to marginal cost c_i and solving for q_{it} yields

$$q_{it}(Q_t) = \min\{k_{it}\,,\ [c_i - D(Q_t)]/D'(Q_t)\}. \tag{2.2}$$

The slope of $q_{it}(Q_t)$ is

$$-1 - q_{it}D''(Q_t)/D'(Q_t) \tag{2.3}$$

if $0 < q_{it}(Q_t) < k_{it}$; otherwise, it is zero. A necessary condition for short run equilibrium is that

$$\sum_i q_{it}(Q_t) - Q_t = 0. \tag{2.4}$$

From (2.3), the derivative of the left side of (2.4) is bounded from above by

$$-2 - \sum_i q_{it}D''(Q_t)/D'(Q_t). \tag{2.5}$$

This is less than zero given the assumption of decreasing industry marginal revenue, so there is at most one solution to (2.4). Since the left side of (2.3) is nonnegative when $Q_t = 0$, a solution necessarily exists. This solves the short run analysis given capacity. If all firms have larger capacity the short run equilibrium industry output will be larger, a result used later.

A stationary market structure is one which will persist into the future. Here we will develop necessary conditions for a stationary solution and determine the unique industry structure satisfying them. Suppose a stationary solution exists. Using the superscript s to denote stationary values for a variable, k_i^s must equal q_i^s for otherwise, either the production constraint would not be satisfied or investment could be reduced without violating any production constraints.

Consider the rate of change of discounted future costs at a stationary equilibrium associated with a small change in capacity which is maintained into the future. It is

$$\frac{1}{h'(\delta)} + \sum_{j=1}^{\infty} \beta^j(\delta + c_i) = \frac{1}{h'(\delta)} + \frac{1}{1-\beta}(\delta + c_i) \tag{2.6}$$

as $1/h'(\delta)$ is the cost of new capacity incurred in the current period, δ the additional depreciation costs incurred in all future periods, and c_i the additional variable costs also incurred in all future periods. The rate of change in the present value of revenue is

$$\sum_{j=1}^{\infty} \beta^t[D(Q^s) + k_i^s D'(Q^s)]. \tag{2.7}$$

Using the fact that $(1 - \beta)/\beta = r$ and that equality of (2.6) and (2.7) is necessary for present value maximization, the following necessary condition for long run equilibrium is obtained:

$$q_i^s(Q^s) = \max\left(0, \frac{r/h'(\delta) + \delta + c_i - D(Q^s)}{D'(Q^s)}\right). \tag{2.8}$$

By an argument identical to the one used to prove existence of a unique short run equilibrium except that the long run marginal cost $c_i + \delta + r/h'(\delta)$ must be substituted for the short run marginal cost c_i, existence and uniqueness of a set of q_i^s for $i = 1,..., n$ which satisfy (2.8), can be established. Therefore, if a stationary equilibrium exists it is given by this set of q_i^s. In subsequent sections, it will be shown that a stationary equilibrium structure exists and that any equilibrium path for the market structure converges to this unique stationary solution.

The right side of (2.8) multiplied by $D'(Q^s)$ is just the economic profits m_i per unit of output for firm i. The long run equilibrium share is

$$q_i^s \Big/ \sum q_j^s = m_i \Big/ \sum m_i\,. \tag{2.9}$$

This is indeed a simple relationship which can be tested using market share data and estimates of profits per unit output in excess of a normal return on investment. If initially there are increasing returns to scale, as is surely the case for the automobile industry, the m_i should be interpreted as the long run marginal economic profits per unit.

3. The Problem Facing a Single Firm

The output sequence which maximizes expected present value for firm i, denoted by $\mathbf{q}_i$, is a function of the expected outputs of its rivals $\mathbf{e}_i$. At time t the supremum of discounted future cash flows over all possible future sequences depends only upon k_{it} and expected future outputs of rivals. Letting $v_{it}(k_{it})$ be this value function, by the principle of optimality

$$v_{it}(k_{it}) = \sup_{q_{it}, k_{i,t+1}} \{D(e_{it} + q_{it})\, q_{it} - k_{it} g(k_{i,t+1}/k_{it}) + \beta v_{i,t+1}(k_{i,t+1})\}, \quad (3.1)$$

where q_{it} and $k_{i,t+1}$ are subject to the production constraints implied by (1.1) and (1.3). In order to simplify notation this relationship is denoted by

$$v_{it} = M_{it} v_{i,t+1} \qquad \text{for} \quad t = 0, 1, 2, \ldots . \quad (3.2)$$

Let V be the normed linear space of functions on the interval $(0, \bar{k}]$ with norm

$$\| v \| = \sup_{k \in (0, \bar{k}]} | v(k) | < \infty$$

and V' be the subset of V such that

$$V' = \{v : v \in V \qquad \text{and} \qquad 0 \leqslant v(k) \leqslant \bar{k}\, D(0)/(1 - \beta)\}.$$

As cash flows in any period clearly are bounded by maximum price $D(0)$ times maximum output $\bar{k}$, the v_{it} belong to V'.

The M_{it} operators map V into V and have the following properties:

(P1) For $v, w \in V$, $v \geqslant w$ implies $M_{it} v \geqslant M_{it} w$.

(P2) For constant function γ and $v \in V$, $M_{it}(v + \gamma) = M_{it} v + \beta\gamma$.

(P3) For $v, w \in V$, $\| M_{it} v - M_{it} w \| \leqslant \beta \| v - w \|$.

(P4) If $v \in V'$, then $M_{it} v \in V'$.

(P5) If $v \in V$ is concave, then $M_{it} v$ is concave.

Proof of (P1) and (P2) follow trivially from the definition of M_{it}. Property 4 follows from (P1) and (P2) by an argument of [2, p. 232]. Results (P4) and (P5) have been shown in [8].

The infinite sequence of functional equations have a unique solution sequence with elements all belonging to V'. Let v_i^j be the element of V' obtained by setting $v_{i,t+j} = 0$ and using (3.2) recursively j times to obtain v_{it}^j. By properties P1 and P4, the $\{v_{it}^j\}$ will constitute an increasing sequence of functions belonging to V'. Therefore, their limit v_{it} exists and belongs to V' proving the existence of a solution sequence. To prove uniqueness, let v_{it} and w_{it} be two solution sequences both of which have all elements belonging to V'. By P3, (3.2), and mathematical induction,

$$\| v_{it} - w_{it} \| \leqslant \beta^j \| w_{i,t+j} - v_{i,t+j} \| \leqslant \beta^j\, \bar{k}\, D(0)/(1 - \beta).$$

As j is arbitrary, $w_{it} = v_{it}$ for all t proving there is a unique solution to (3.2), namely the limit of the $\{v_{it}^j\}$ sequences.

Given (P5), the v_{it}^j functions are all concave as the zero function is trivially concave, and v_{it}^j was obtained by successive operations of $M_{i,t+k}$ for $k = j - 1, j - 2,..., 1, 0$ on the zero function. Thus, the v_{it} are concave.

From (3.1), the optimal $k_{i,t+1}$ given k_{it} must satisfy

$$g'(k_{i,t+1}/k_{it}) = \beta v'_{i,t+1}(k_{i,t+1}). \tag{3.3}$$

The left side of (3.3) is strictly increasing given the strict convexity of g, while the left side given convexity of $v_{i,t+1}$ is decreasing; thus, the curves are as pictured in Fig. 1.[4] Equation (3.3) defines implicitly a relationship which the $\mathbf{k}_i$ sequence must satisfy

$$k_{i,t+1} = f_{it}(k_{it}). \tag{3.4}$$

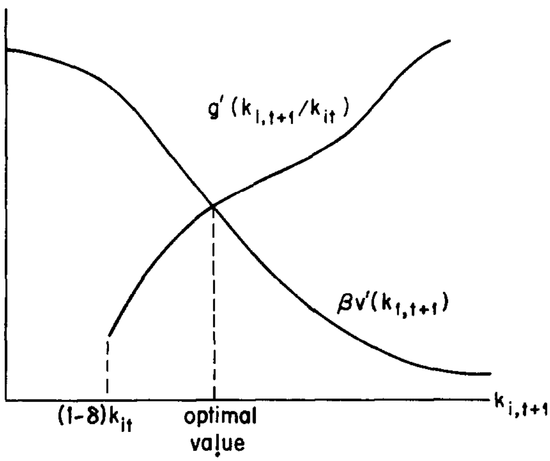

FIGURE 1.

The larger k_{it} the larger $k_{i,t+1}$ for increase in k_{it} shift the curve $g'(k_{i,t+1}/k_{it})$ down moving the intersection in Fig. 1 to the right.[5]

LEMMA. *If* $\mathbf{e}_i^a \geqslant \mathbf{e}_i^b$ *(that is,* $e_{it}^a \geqslant e_{it}^b$ *for all* t*) then* $f_{it}^a \leqslant f_{it}^b$ *uniformly in* k *for all* t.

Proof. From (3.1) if e_{it} is larger and $v'_{i,t+1}$ uniformly larger, then v'_{it}

[4] The concavity of the $v_{i,t+1}$ implies $v'_{i,t+1}$ exists. If $v'_{i,t+1}$ is not continuous and as a result no $k_{i,t+1}$ satisfies (3.3), then the optimal $k'_{i,t+1}$ has the property that for any $\epsilon > 0, g'[(k_{i,t+i} + \epsilon)/k_{it}] > \beta v'_{i,t+1}(k_{i,t+1} + \epsilon)$ and $g'(k_{i,t+1} - \epsilon)/k_{it}) < \beta v'_{i,t+1}(k_{i,t+1} - \epsilon)$.

[5] By larger we mean greater than or equal to while strictly larger means greater than. Similar conventions hold for increasing versus strictly increasing, concave versus strictly concave, etc.

is uniformly larger. This along with a successive approximation argument proves the result.

Equation (3.4), given an initial k_{i0} and expectation sequence for rival outputs $\mathbf{e}_i$, may be used to determine the unique path for capacity that maximizes expected present value. The relationship is denoted by

$$\mathbf{q}_i = T_i(\mathbf{e}_i). \tag{3.5}$$

A more direct argument that this dynamic programming approach could have been used to establish the existence and uniqueness of such a function. One merely shows that the present value function over sequences, $\mathbf{q}_i$ is continuous in the product topology and the function is strictly concave. Considering the compact set of sequences such that for all t, $0 \leqslant q_{it} \leqslant \bar{k}$, uniqueness and existence of an optimal $\mathbf{q}_i$ conditional on $\mathbf{e}_i$ is established. As the present value function is jointly continuous in $\mathbf{q}_i$ and $\mathbf{e}_i$, the mapping T_i is continuous in the product topology. This was not done because of the need to establish properties of the $f_{it}(k_{it})$ functions which will be used to characterize the equilibrium path.

If the e_{it} are all equal to, say, $e_i{}^*$, then the f_{it} are all equal to f_i. The monotonicity of f_i implies the optimal path of k_{it} will be monotonic which in turn implies that it will converge to some value, say $q_i{}^*$. By the argument used to develop the necessary condition for a stationary solution in section 2, the $q_i{}^*$ must satisfy

$$q_i{}^* = \Phi_i[D(e_i{}^* + q_i{}^*],$$

where

$$\Phi_i[D(y)] = \max\{0, [r/h'(\delta) + c_i + \delta - D(y)/D'(y)]\}.$$

If $k_{i0} = q_i{}^s$ and $e^s_{it} = \sum_{j \neq i} q_j{}^s$ for all i, each firm will produce $q_i{}^s$ in every period and expectations will be satisfied; that is

$$\sum_{i \neq j} T_j(e_i{}^s) = e_i{}^s.$$

This proves a stationary industry structure does in fact exist.

Letting lim inf denote limit inferior and lim sup limit superior, the lemma implies

$$\limsup q_{it} \leqslant \Phi_i[D(\limsup q_{it} + \liminf e_{it})] \leqslant \Phi_i(\limsup D_t) \tag{3.7}$$

and

$$\liminf q_{it} \geqslant \Phi_i[D(\liminf q_{it} + \limsup e_{it})] \geqslant \Phi_i(\liminf D_t). \tag{3.8}$$

In the above $D_t = D(q_{it} + e_{it})$ is the expected market price in period t. Relationships (3.7) and (3.8) imply if $\mathbf{e}_i$ converges then both $\mathbf{q}_i$ and the expected price sequence $\mathbf{D}$ also converge.

4. Existence of An Equilibrium Path

In the previous section, it was established that firm i output sequence $\mathbf{q}_i$ is a well defined function of its expectation of rivals output.

Let

$$\mathbf{Q}_i(\mathbf{e}_i) = \sum_{j \neq i} T_j(\mathbf{e}_j) \qquad \text{for} \quad i = 1,..., n. \tag{4.1}$$

Let L be the normed linear space of n-tuplets of infinite sequences with norm

$$\| \mathbf{e} \| = \sum_{i,t} \beta^t \mid e_{it} \mid < \infty \tag{4.2}$$

for $\mathbf{e} \in L$ and B be the subset of L such that

$$B = \{\mathbf{e} : \mid e_{it} \mid \leqslant \bar{k} \qquad \text{for all} \quad i, t\}. \tag{4.3}$$

Relationship (4.1) is a mapping of B into itself. The set B is convex and compact while (4.1) is a continuous mapping on B, given the norm. By the Tychanoff theorem [5, p. 414] this mapping will have a fixed point; that is a set of $\mathbf{Q}_i$ belonging to B such that

$$\mathbf{Q}_i = \sum_{j \neq i} T_j(\mathbf{Q}_i) \qquad \text{for} \quad i = 1,..., n.$$

The set of output sequences $\mathbf{q}_i = T_i(\mathbf{Q}_i)$ and expectations $\mathbf{e}_i = \mathbf{Q}_i$ will be an equilibrium path because each firm maximizes its present value given its expectations and each firm's expectations are realized.

5. Convergence to the Stationary Solution

The results of this section can be summarized by the following theorem.

Theorem. *Any price sequence associated with an equilibrium, say*

$$\left\{D_t = D\left(\sum_i q_{it}\right)\right\}$$

converges to the stationary solution price

$$D^s = D\left(\sum_i q_i^s\right)$$

and the output sequence $\mathbf{q}_i$ *to* q_i^s *for all i.*

Proof. By the results of Section 3

$$\liminf D_t \leqslant D^s \leqslant \limsup D_t ,$$

so if D_t converges then it must converge to D^s.

Suppose that D_t does not converge which implies $\limsup D_t > D^s$. Let $\{t(j)\}$ be a subsequence such that $D_{t(j)}$ converges monotonically to $\limsup D_t$ and $D_{t(j)} > D_{t(j)+1}$ for all j. For sufficiently large J, $q_{it(j)} = k_{it(j)}$ for $j \geqslant J$; that is, firms will be producing at capacity. To simplify notation $t(j)$ will be denoted by t in the remainder of this section.

For any j, the firms can be categorized into the following three sets:

$$\begin{aligned} &S_1 \quad \text{if} \quad k_{it} > k_{i,t+1} , \\ &S_2 \quad \text{if} \quad k_{i,t-1} < k_{it} \leqslant k_{i,t+1} , \\ &S_3 \quad \text{if} \quad k_{i,t-1} \geqslant k_{it} \leqslant k_{i,t+1} . \end{aligned}$$

The nature of this proof is to show S_2 and S_3 are empty sets. This implies all firms will have smaller capacities (in the limit) in period $t + 1$ than in t. By the results of Section 3, if all firms have larger capacity, the price will be smaller implying $D_{t+1} \geqslant D_t$. But, $D_{t+1} < D_{t+1}$ by constructing, which provides the contradiction.

Consider first S_3. Since $f_{i,t-1} \geqslant f_{it}$ and these functions are increasing, $k_{i,t-1} \geqslant k_{it}$ implies $f_{i,t-1}(k_{i,t-1}) = k_{it} \geqslant f_{it}(k_{it}) = k_{i,t+1}$. This is not possible proving S_3 is empty.

Consider next set S_2 firms for which $k_{it} \geqslant k_{i,t-1}$ and $k_{i,t+1} > k_{it}$. The rate of change in present value π_i, divided by β^{t-1}, of changing k_{it} while holding other capacities fixed is

$$\frac{d\pi_i}{dk_{it}} = -\beta^{-1} g'\left(\frac{k_{it}}{k_{i,t-1}}\right) - g\left(\frac{k_{i,t+1}}{k_{it}}\right) + \frac{k_{i,t+1}}{k_{it}} g'\left(\frac{k_{i,t+1}}{k_{it}}\right)$$

$$+ D_t + D_t' k_{it} - c_i , \qquad (5.3)$$

where $D_t' = D'(\sum_i q_{it})$. Given the assumed properties of g, namely it is an increasing continuously differentiable strictly convex function with $g(1) = \delta$, it follows

$$\frac{d\pi_i}{dk_{it}} \geqslant -\beta^{-1}g'(1) - \delta + g'(1) - c_i + D_t + D_t' k_{it}$$
$$= -\frac{r}{h'(\delta)} - \delta - c_i + D_t + D_t' k_{it}. \tag{5.4}$$

The expression on the right will be positive for sufficiently large j unless $k_{it} \geqslant \Phi_i(\limsup D_t)$. But, by (3.7)

$$\limsup k_{it} < \Phi_i(\limsup D_t),$$

proving S_2 is also empty.

Because the D_t converges to D^s, by (3.7) and (3.8) q_{it} must converge to q_i^s proving the theorem.

6. Concluding Comments

Our definition of equilibrium path is the obvious generalization of Cournot's equilibrium analysis [4]. Some may criticize the use of the quantity sequence as the decision variable as Bertrand [1] criticized the work of Cournot. Its use is needed to avoid the knife-edge instability of Bertrand's noncooperative price setting model. If, however, there is product differentiation and cross-elasticities of demand are finite, it does not matter whether price or quantity is the decision variable. Other solution concepts besides noncooperative behavior are of interest. For example, suppose one firm, the dominant one, takes into consideration the effect of his actions upon rivals—namely that rivals will react to his output increases by decreasing theirs. The dynamic implication of such an assumption is an interested question in need of analysis.

The policy implications of the analysis are not as clear as they may seem. General Motors enjoys higher profit per unit output than does its rival auto producers. Given the assumptions of our analysis, breaking this company into two low cost producers would result in a reduction in market price and a reduction in average production cost for the industry. Thus it seems such an antitrust action would be in the public interest. If, however, General Motors realizes that they will be broken up if they obtain more than a given market share which happens to be less than their stationary noncooperative share, it is in their interest to produce less than the noncooperative stationary output. In this way, they avoid antitrust action, the public pays higher prices, and average costs of production in the industry are higher—results *not* in the public interest.

Uncertainty could have been introduced in the form of randomness in the demand function or short run marginal costs c_i, provided the disturbances were independent between periods. Assuming increasing short run marginal costs rather than assuming they were constant up- to capacity and infinite beyond is a straigtforward extension. It, however, would be necessary that short run marginal costs depend upon q_{it}/k_{it} to preserve the constant returns to scale assumption in the long run.

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