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EFFECTIVE QUARK MASSES IN THE CHIRAL LIMIT *

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The behavior of effective, scale dependent quark masses in the chiral limit is studied using the operator product expansion. Their relation to current and constituent quark masses is clarified. Current correlation functions are examined as an example of separating effects of spontaneous and explicit chiral symmetry breaking.

1. Introduction

A scale dependent, effective quark mass, $m(Q)$ was introduced in the field-theoretic study of inclusive lepton-hadron scattering [1]. It has the interpretation of the parton mass appropriate to the incident momentum transfer, and it also interpolates between current and constituent quark masses. However, the analysis in ref. [1] is based on perturbation theory. This is likely to be reliable for small coupling constant except for effects which vanish to all orders but are known to be non-zero. A case in point is effective quark masses in the chiral limit of colored quark gauge theory.

An operator product expansion [2] (OPE) will show that the contribution of dynamical, spontaneous chiral symmetry breaking to $m(Q)$ goes like (v/Q^2) $(\log Q^2)^{-15/27}$ for large Q, where $v = \langle 0|\bar{\psi}\psi|0\rangle$, for an SU(3) color gauge theory with three massless quark flavors. If there is also an explicit breaking of chiral symmetry due to a bare mass, m_{bare}, then there is an additional contribution to $m(Q)$ that goes like $m_0(\log Q^2)^{-12/27}$, where $m_0 \propto m_{\text{bare}}$. Hence spontaneous effects vanish rapidly relative to any explicit symmetry breaking at short distances.

2. Outline

First, there is a review of the definition and significance of $m(Q)$.

Next, aspects of the problem of spontaneous symmetry breaking are discussed.

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Two examples are mentioned where spontaneous effects are calculable and do *not* vanish relative to explicity symmetry breaking at short distances. In the end, these results are shown to be consistent with the OPE analysis.

The OPE is applied to the quark propagator in a chiral symmetric quark-gluon model. Perturbation theory is argued to be good for coefficient functions, while the (as yet) uncalculable dynamical symmetry breaking resides in vacuum expectations of local operators. This argument yields the advertised result for $m(Q)$. It is then repeated in the presence of an explicit quark mass term.

Methods of estimating the relative importance of the two effects for finite $Q^2 < \infty$ are discussed. As an example of the utility of these ideas, vector and axial vector current correlation spectral functions are calculated.

3. The effective quark mass

$m(Q)$ is defined via the complete, renormalized quark propagator for spacelike momentum q:

$$\int e^{iqx} d^4x \, \langle 0|T(\psi(x)\, \bar{\psi}(0))|0\rangle = \frac{i}{A\not{q} - B} \,, \tag{1}$$

where A and B are functions of $Q^2 = |q^2|$; M, the renormalization mass; the renormalized gauge coupling, $g(M)$; and the renormalized quark masses. A and B define $m(Q)$ by

$$m(Q) \equiv \frac{B(Q^2)}{A(Q^2)} \,. \tag{2}$$

Even though $m(Q)$ is defined by reference to a spacelike q, it plays the role of the struck parton mass in the OPE analysis of lepton-hadron scattering. This is because $m(Q)$ is the most appropriate mass parameter to use when computing the operator coefficient functions in the expansion of two currents [1]. The virtue of computing in terms of $m(Q)$ is that potentially large logarithms that appear when using other conventions are automatically summed. The coefficient functions depend on the spacelike momentum transfer. Moments of the physical structure functions are obtained via analytic continuation in the energy transfer $\nu = E - E'$ [3].

To lowest order in perturbation theory, $m(Q)$ satisfies

$$Q\frac{dm(Q)}{dQ} = -m(Q)\, g^2(Q)\, a \,, \tag{3}$$

with

$$a = \frac{1}{2\pi^2}\left[1 - \frac{m^2}{Q^2}\log\left(1 + \frac{Q^2}{m^2}\right)\right] \approx \frac{1}{2\pi^2}\left[\frac{1}{1 + 2\dfrac{m^2}{Q^2}}\right] . \tag{4}$$

If we distinguish different quark flavors with a subscript $i (i = \mathrm{u,d,s})$, then the m's in eqs. (3) and (4) should be m_i's (nowhere summed). The theory with three quark flavors is asymptotically free [4], so $g^2(Q)$ goes like $1/\log Q^2$ as $Q^2 \to \infty$. Since a is flavor independent for $Q^2 >> m_i^2$, the $m_i^2(Q)$ vanish like the same negative power of $\log Q^2$ for each i. Hence, they approach fixed ratios as $Q^2 \to \infty$.

It is trivial to show that *in perturbation theory*

$$\lim_{Q \to \infty} \frac{m_i(Q)}{m_j(Q)} = \frac{m_{i,\mathrm{bare}}}{m_{j,\mathrm{bare}}} . \tag{5}$$

The central point of this paper is to show that even in the presence of dynamical spontaneous chiral symmetry breaking, undetectible in quark-gluon perturbation theory, eq. (5) is true. The proof will rest on asymptotic freedom and the scale dimensions of the fundamental fields. Hence, eq. (5) will not be true for all field theories.

The significance of eq. (5) is that the ratio of bare masses determines the symmetry properties of the quark-gluon Hamiltonian. So it is these ratios that enter symmetry breaking calculations in current algebra [5]. For example, from the kaon and pion masses, one may deduce that $(m_\mathrm{u} + m_\mathrm{d})_\mathrm{bare}/2m_{\mathrm{s,bare}} \simeq \frac{1}{25}$.

The connection of $m_i(Q)$ to constituent masses is somewhat less direct and is precise only for heavy quarks. For light quarks, the analysis is qualitative. The connection is made via the smeared $\mathrm{e^+e^-}$ cross section. Smearing the $\mathrm{e^+e^-}$ cross section in energy is equivalent to computing the hadronic vacuum polarization for complex energies, slightly above and slightly below the real axis [6]. If the energy is sufficiently high that the effective color coupling is small, the smeared $\mathrm{e^+e^-}$ cross section will have a threshold for each new heavy quark centered around $E_\mathrm{threshold} = 2m_i(E_\mathrm{threshold})$. From the behavior of the charmed quark contribution to $\mathrm{e^+e^-}$ annihilation and the location of the J, it is plausible that the lightest $\mathrm{q}_i\bar{\mathrm{q}}_i$ vector meson lies approximately at the threshold for $\mathrm{q}_i + \bar{\mathrm{q}}_i$ production (in the smeared cross section). Hence for heavy quarks, it makes sense to define the constuent quark mass, $m_i^\mathrm{constituent}$, by the equation

$$m_i^\mathrm{constituent} \equiv m_i(Q = 2m_i^\mathrm{constituent}) . \tag{6}$$

Light quarks are quarks such that $g^2(Q = 2m_i(Q))$ is large. Hence, the location of the first vector meson and the appropriate threshold in smeared $\mathrm{e^+e^-}$ annihilation are effected by the color interactions. However, eq. (6) still makes qualitative sense in that crude estimates using eq. (6) [1] give $m_\mathrm{u,d}^\mathrm{constituent} \sim 350–400$ MeV, which is just what is commonly meant by constituent masses.

4. Problems with spontaneous symmetry breaking

According to eq. (3), $m(Q) \to 0$ as $Q \to \infty$. So at large Q we may wish to think of $m(Q)$ as a small perturbation on a chiral symmetric theory. But what about spontaneous breaking of chiral symmetry? Will that alter eq. (5)? Consider a world in which $m_{u,bare} = 0 \neq m_{s,bare}$, i.e., $m_{u,bare}/m_{s,bare} = 0$. According to eqs. (3) and (5), we would conclude $m_u(Q)/m_s(Q) = 0$ for all Q. Yet due to spontaneous breaking of chiral SU(2) × SU(2), the u quark has a non-zero constituent mass. If eq. (5) is correct, then something is wrong with eq. (3). Even correcting eq. (3) to all orders in perturbation theory will not help because in perturbation theory (of quarks and color gluons) $Q\, dm/dQ$ is always proportional to $m(Q)$.

To show that this not a completely trivial problem, I will mention two models in which spontaneous effects are calculable and eq. (5) is wrong.

The first example is massless, N component fermions in two space-time dimensions interacting via a chiral symmetric, renormalizable, four-fermi coupling. When studied in the leading $1/N$ approximation, this model exhibits spontaneous breakdown of chiral symmetry [7]. To leading order in $1/N$, the fermion propagator is $(\not{q} - M_F)^{-1}$, where M_F is the single free parameter of the theory. So $m(Q) = M_F$ while $m_{bare} = 0$. If a small bare mass were added as a perturbation, it would simply add to M_F.

The second example is in four dimensions. Consider a chiral symmetric model with fundamental scalar fields as well as quarks and gluons, studied in perturbation theory and set up so that the vacuum is asymmetric. Then add small bare masses. $Q\, dm/dQ$ would receive contributions proportional to the scalar vacuum expectation as well as the bare mass. When integrated, both these terms would give contributions to $m(Q)$ that go like powers of log Q. But which dominates for $Q \to \infty$ is totally model dependent. In some models the spontaneous effects may vanish relative to explicit masses, and *vice versa.*

5. OPE analysis

The proposed remedy for this confusion is to study the product $\psi(x)\, \overline{\psi}(0)$ with a short distance expansion. This is in direct analogy with the work of Bernard et al. [8] who used the OPE and asymptotic freedom to separate spontaneous from explicit symmetry breaking effects on spectral function sum rules [9, 2]. The utility of this program rests on a theorem proven in ref. [8]. The theorem states that the coefficient functions in the OPE manifestly reflect the symmetries of the Lagrangian, i.e., whether or not they are symmetries of the vacuum. In particular, if the Lagrangian is chiral invariant, then the coefficient functions transform linearly, as appropriate for the relevant operators. So there is no outstanding objection to computing the coefficient functions for spacelike momentum in perturbation theory (if the effective coupling is small) since perturbation theory does give the correct chiral transformation properties. *All spontaneous symmetry breaking resides in the matrix elements of the local operators.* These matrix elements may not be calculable, but they could be deduced from experiment or crudely estimated by dimensional analysis.

Consider first the OPE in chiral symmetric, massless colored-quark-gluon gauge theory,

$$\int e^{iqx}\, d^4x\, \langle 0|-iT(\psi(x)\,\bar{\psi}(0))|0\rangle = \sum_n c_n(q)\langle 0|O^n|0\rangle \tag{7}$$

$$= c_1(q)\,\langle 0|1|0\rangle + c_2(q)\,\langle 0|\bar{\psi}\psi|0\rangle + \dots \, .$$

The operator $\bar{\psi}\psi$ is a Lorentz scalar and color singlet (assuming quarks are confined by the maintenance of exact color symmetry). The ... stands for terms with operators of higher dimension. Eq. (7) is represented in fig. 1. For Q^2 large enough that $g^2(Q)$ is small, the $c_n(q)$ behave as canonical powers of Q times calculable logarithms. So the operators of higher dimension have coefficients that are down by negative powers of Q.

If the operators O^n, ψ, and $\bar{\psi}$ are renormalized at a fixed scale M, then the renormalization group analysis implies that the c_n have the form

$$c_n(q) = c_n(q;g^2(Q),Q)\exp\left\{\int_M^Q \gamma_n \frac{dQ'}{Q'}\right\}, \tag{8}$$

where

$$\gamma_n = \gamma_n(g^2(Q')) = 2\gamma_\psi + \gamma_{O^n} \tag{9}$$

with γ_ψ and γ_{O^n} the anomalous dimensions of the fermi field and the O^n.

The anomalous dimension of the operator 1 is zero and $c_1(q;g^2(Q),Q) \equiv 1/\not{q}$. Using the definition of $m(Q)$ in eqs. (1) and (2),

$$m(Q) = c_2(q;g^2(Q),Q)\exp\left\{\int_M^Q \gamma_{\bar{\psi}\psi}\frac{dQ'}{Q'}\right\}\langle 0|\bar{\psi}\psi|0\rangle + \dots\, , \tag{10}$$

where again the ... stands for terms that vanish by additional powers of $1/Q^2$.

To $O(g^2)$, $\gamma_{\bar{\psi}\psi} = g^2 2\pi^2$, as computed from the graph in fig. 2, and

$$c_2(q;g^2(Q),Q) = i\delta^4(q) + \frac{4g^2(Q)}{Q^4} \tag{11}$$

(in Landau gauge) for an SU(3) of color. See fig. 3. The δ-function term does not

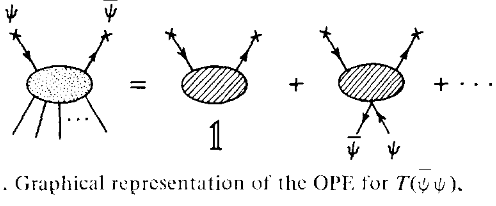

Fig. 1. Graphical representation of the OPE for $T(\bar{\psi}\psi)$.

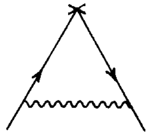

Fig. 2. The lowest order contribution to $\gamma_{\bar{\psi}\psi}$.

contribute to the connected quark propagator. Combinding eqs. (10) and (11) at high Q yields

$$m(Q) \approx \frac{4g^2(Q)}{Q^2} \left(\frac{g^2(Q)}{g^2(M)}\right)^{-d} \langle 0|\bar{\psi}\psi(M)|0\rangle \,, \tag{12}$$

where $d = 12/27$ and $g^2(Q) = 48\pi^2/27 \log (Q^2/\Lambda^2)$ for three quark flavors. Λ is the free parameter that sets the scale of the color coupling. For four quark flavors, replace 27 by 25 in the above relations. The flavor index is suppressed on $m(Q)$ and $\bar{\psi}\psi(M)$.

It is now a straightforward matter to include explicit chiral symmetry breaking *via* mass terms. The prescription is just as before: the coefficient functions are computed in perturbation theory but now including explicit quark masses in the Feynman rules. It is convenient to renormalize the perturbation theory as suggested in ref. [1]. For example, define mass and wave function renormalizations by

$$S_F^{-1}(q)\Big|_{q^2=-M^2} \equiv \not{q} - m_0(M) \,, \tag{13}$$

where S_F is the renormalized quark propagator in perturbation theory; M sets the scale of all renormalizations and $m_0(M)$ is the renormalized quark mass. Hence, it is $m_0(Q)$ that satisfies eq. (3). However, the full $m(Q)$ in analogy to eq. (12) for

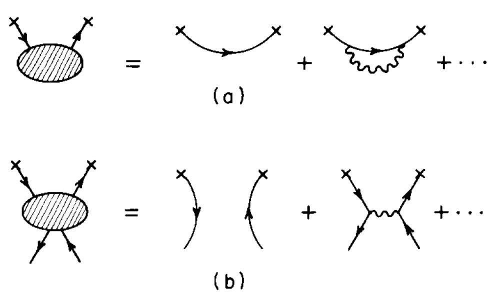

Fig. 3. Diagrams contributing to the coefficients of 1 (a) and $\bar{\psi}\psi$(b) in the expansion of $T(\psi(x)\,\bar{\psi}(0))$.

large Q is

$$m(Q) \approx m_0(M) \left(\frac{g^2(Q)}{g^2(M)}\right)^d + \frac{4g^2(Q)}{Q^2}\, v(M) \left(\frac{g^2(Q)}{g^2(M)}\right)^{-d} , \tag{14}$$

where I have dropped effects of $O(m_0^2/Q^2)$ that enter $\gamma_{\overline{\psi}\psi}$, which governs $m_0(Q)$ and $v(Q)$ (see eq. (4)), and β_g, which governs $g^2(Q)$ [1].

The explicit mass terms may be different for different flavors. The ratios of the $m_i(Q)$ approach the ratios of the $m_{0,i}(Q)$ for large Q^2, which themselves go to the ratios of bare masses. Hence eq. (5) is true even including spontaneous effects in quark-gluon theory. And the chiral limit is smooth in that as $m_{\mathrm{u,bare}}/m_{\mathrm{s,bare}}$ goes to zero, so will $m_{\mathrm{u}}(Q)/m_{\mathrm{s}}(Q)$ for large Q; however $m_{\mathrm{u}}(Q)/m_{\mathrm{s}}(Q)$ need not be small at low Q.

The explanation of the two counter examples raised against eq. (5) rests simply on dimensional analysis and the OPE. In the two-dimensional model, $\overline{\psi}\psi$ is of dimension one and can compete with an explicit m_0. Likewise, in the four-dimensional model, the fundamental scalar field has the same dimension as m_0.

6. Using effective masses

Turning these ideas into useful phenomenology will require some guesswork, new hypotheses, and experimental imput because it is impossible to measure power dependence in Q as Q literally goes to infinity. We need estimates of the relative sizes of the two terms in eq. (14) at finite Q. What follows is a variety of arguments to estimate m_0 and v. I will then give a sample application.

Light quark constituent masses and low Q phenomenology are presumably roughly the same whether chiral SU(2) × SU(2) is exact or slightly broken. This suggests that $4g^2(Q)\, v(Q)/Q^2$ is something like 300 MeV for $Q \sim 700$ MeV. Of course, this is taking eqs. (7) and (12) far too seriously at low Q, but I only want an order of magnitude estimate. If the only free parameter of the theory is Λ, one would expect the constituent masses to turn out of order Λ. Λ also presumably sets the size of hadrons and the transverse momentum cutoff. In recent phenomenological analyses [10, 11] $400 \lesssim \Lambda \lesssim 700$ MeV was obtained. So the estimates hold together.

There is no compelling reason to think of flavor SU(3) (or SU(n)) symmetry breaking as spontaneous; so v is approximately flavor symmetric. Consequently v does not enter significantly into the constituent masses of heavy quarks. For example, for the charmed quark $m_{\mathrm{c}}(3\text{ GeV}) \approx 1.5\text{ GeV} \approx m_{0,\mathrm{c}}(3\text{ GeV}) + (0.3\text{ GeV})(0.7/3)^2\,(0.6)$. So $m_{\mathrm{c}}(3\text{ GeV}) \simeq m_{0,\mathrm{c}}\,(3\text{ GeV})$.

The strange quark is an intermediate case. If we take $m_{\mathrm{s}}^{\mathrm{constituent}} = 500$ MeV, then eq. (14) suggests that there is a small but non-negligible spontaneous contribution, e.g. $500\text{ MeV} = m_{\mathrm{s}}\,(1\text{ GeV}) = m_{\mathrm{s},0}(1\text{ GeV}) + 125\text{ MeV}$ or $m_{\mathrm{s},0}\,(1\text{ GeV}) = 375$ MeV.

The ratio $m_{\rm p,bare}/m_{\rm s,bare} \approx \frac{1}{25}$, obtained from $m_\pi^2/m_{\rm k}^2$, determines $m_{\rm p,0}(Q)/m_{\rm s,0}(Q)$; so $m_{\rm p,0}$ (1 GeV) $\approx$ 15 MeV. For Λ = 500 MeV, $M_{\rm p,0}(Q)$ grows to about $m_{\rm p,0}$ (700 MeV) = 17 MeV at Q = 700 MeV. So as expected, the light quark constituent masses come virtually from spontaneous symmetry breaking. Also, with these estimates of $m_{\rm p,0}$ and v, $m_{\rm p,0}(Q)$ contributes less than half of $m_{\rm p}(Q)$ for $Q < 3$ GeV. Of course, $m_{\rm p,0}(Q)$ dominates $m_{\rm p}(Q)$ as $Q \to \infty$.

As an example of the utility of effective quark masses, consider current correlation spectral functions, ρ defined by

$$\int e^{iqx}\, d^4x\, \langle 0|T(J(x)\, J^\dagger(0))|0\rangle = \int_0^\infty d\mu^2\, \rho(\mu^2)\, \frac{i}{q^2 - \mu^2 + i\epsilon}\,, \tag{15}$$

where the Lorentz structure is suppressed. The application of renormalization-group-improved perturbation theory has been studied in some detail for the case of two electromagnetic currents, for which ρ is proportional to the hadronic e^+e^- annihilation cross section [6, 12].

The renormalization group improvement is the relation

$$\rho(Q^2; g(M), m(M), M) = \rho(Q^2; g(Q), m(Q), Q)\,, \tag{16}$$

where $m(M)$ stands for all masses renormalized with reference to some scale M, e.g. as in eq. (13). Even when $g(Q)$ is small, perturbation theory may be terrible for $\rho(Q^2)$ taken literally, i.e., locally in Q. However, perturbation expansions should be good for ρ if smeared or integrated in Q [6].

If $m(Q)$ in eq. (16) is interpreted as the complete, effective quark mass as defined by eq. (2), then the improved perturbation expansion for ρ reproduces the leading terms to first order in spontaneous symmetry breaking of the straight forward OPE analysis of $\langle 0|T(J(x)\, J(0)|0\rangle$ as given in ref. [8]. The dependence on explicit symmetry breaking is the same in both analyses. To second order in spontaneous breaking, eq. (2) and perturbation theory for eq. (16) agree with ref. [8] only if $\langle 0|\bar\psi\psi\bar\psi\psi|0\rangle \simeq \langle 0|\bar\psi\psi|0\rangle\, \langle 0|\bar\psi\psi|0\rangle$. And in general, higher orders in spontaneous breaking will introduce new operators. The present prescription is based on the estimate that their vacuum expectations are roughly given by the appropriate power of $\langle 0|\bar\psi\psi|0\rangle$. The counting of powers of q^2 is the same in any case. While no great theoretical advance, the representation of ρ in terms of $m(Q)$ may be a modestly helpful mnemonic because ρ is given by the graphs of fig. 4 with the prescription that the quark masses are taken to be $m_i(Q)$, with Q^2 as the current momentum squared.

If $J_\mu(x)$ is a quark current like $\bar q\, \gamma\mu(1 \text{ or } \gamma_5)\, q'$, then

$$\mathrm{Im} \int \langle 0|T(J_\mu J_\nu^\dagger)|0\rangle\, e^{iqx} d^4x$$

is proportional to

$$-\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\rho_{\rm T} + \frac{q_\mu q_\nu}{q^2}\, \rho_{\rm L}\,. \tag{17}$$

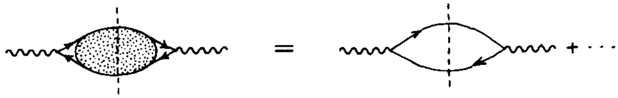

Fig. 4. The leading contribution to the spectral functions.

The lowest order contribution to ρ_T and ρ_L can be computed from fig. 4. One finds.

$$\rho_T = \phi \left[1 \pm \frac{2mm'}{q^2} - \frac{(m-(\pm m'))^2}{2q^2}\left(1 + \frac{(m \pm m')^2}{q^2}\right)\right], \qquad (18)$$

$$\rho_L = \frac{3}{2}\phi \frac{(m-(\pm m'))^2}{q^2}\left[1 - \frac{(m \pm m')^2}{q^2}\right],$$

with ϕ, the two-particle phase space

$$\phi = \sqrt{1 - \frac{(m+m')^2}{q^2}}\sqrt{1 - \frac{(m-m')^2}{q^2}}. \qquad (19)$$

The ±'s refer to the combinations vector-vector (V-V) or axial-axial (A-A). The normalization in eq. (18) is given by $\rho_T \to 1$ as $q^2 \to \infty$. Often of interest is the combination

$$\rho_T^{V\text{-}V} - \rho_L^{A\text{-}A} = \rho_L^{A\text{-}A} - \rho_L^{V\text{-}V} = 6\phi \frac{mm'}{q^2}. \qquad (20)$$

Its form is particularly simple because the relevant amplitude is proportional to $mm'\mathrm{tr}(\gamma_\mu \gamma_\nu)$. Now interpret m and m' as the effective quark and quark$'$ masses, evaluated at q^2.

The convergence properties of spectral integrals and linear combinations of spectral integrals can be read off from eqs. (18)–(20) using eq. (14). For example, as $Q^2 \to \infty$, $\rho^{V\text{-}V} - \rho^{A\text{-}A} \propto (q^2)^{-3} (\log q^2)^{-30/27}$ in a chiral symmetric theory, but $\rho^{V\text{-}V} - \rho^{A\text{-}A} \propto (q^2)^{-1} (\log q^2)^{-24/27}$ if both relevant quarks have bare masses.

But eqs. (14), (18)–(20) tell much more than asymptotic properties. They give the local (or smeared) values of the spectral functions as determined in colored-quark-gluon gauge theory (at least in lowest order in q^2). The effects of operators of dimension higher than $\bar{\psi}\psi$ have been ignored, but these become important only at low q^2 where the expansion in power of g^2 is useless. Furthermore, intergrals over low q^2 are determined by these considerations in the sense of finite energy sum rules, as exploited in ref. [10].

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