QCD OFF THE LIGHTCONE AND THE DEMISE OF THE TRANSVERSE MOMENTUM CUT-OFF★

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Criteria are given for the applicability of the asymptotic freedom of QCD. Perturbative QCD has a much broader range of utility than lightcone dominated phenomena. The most striking predictions pertain to transverse momenta that scale, i.e. grow proportionately to the large invariants. While hints of these unlimited transverse momenta are already visible in existing data, the predictions are unmistakable for μ-p, e^+-e^-, and hadron-hadron experiments now planned.

The testable predictions of quark-gluon gauge theory (QCD) are woefully few. Traditionally, these predictions were based on short distance or lightcone arguments, which are not immediately extendable to more complex kinematic configurations. These traditional analyses of inclusive e-p and e^+-e^- collisions were designed to be sufficiently general to apply to wide classes of strongly interacting theories. However, QCD is unique in its asymptotic freedom [1], and the QCD analysis rests on perturbation theory. Since QCD is more specific than the general lightcone analyses, it makes predictions where the latter do not. Under assumptions no more stringent than those needed for the inclusive e-p and e^+-e^- analyses, we can go on to make predictions in a wide variety of situations. I will explain what these situations and assumptions are, but they can be characterized simply: *Use perturbation theory where it is not obviously wrong.*

The criteria for applicability of perturbative QCD are: 1) low orders of perturbation theory are well-behaved, and 2) there are no obvious non-perturbative effects which are larger than the calculable perturbative ones. This analysis rests on the assumptions that 3) the strong Lee-Nauenberg-Kinoshita theorem [2,3] (i.e. for differential cross sections) is true, and 4) the solution of QCD will show that the necessary non-perturbative effects do not spoil these predictions.

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1. Self-consistency of low orders: Smallness of the coupling constant is only a necessary condition for the reliability of perturbation theory. We must also avoid situations where the expansion coefficients are uncontrollably large. This can occur in the form of terms like $\log m^2/M^2$, where m^2 is a quark or gluon mass and M^2 is some large invariant; like $1/v$, where v is the relative velocity of produced quarks above a threshold; and like $\log x$ or $(1-x)^{-1}$, where x is some dimensionless ratio of large invariants which ranges over $0 < x < 1$.

The $\log m^2$ terms invariably arise in straightforward calculations. In many situations these are spurious singularities, as they may be reabsorbed into quark and gluon wave functions [4] (see point #3). We must allow the initial and final quarks and gluons to have variable p^2's corresponding to the range given by the hadron wave functions. Since this range is fixed relative to the large invariants M_i^2, we take $m^2 \neq p^2 \ll M_i^2$. The details of how to reabsorb these $\log(p^2 - m^2)$ terms are given elsewhere [4,5].

The threshold singularities must be smeared over. This amounts to a small excursion into complex energies [6]. How much smearing is necessary is suggested by perturbation theory itself: enough to make higher orders smaller.

Kinematic singularities in the scaling variables are insurmountable. They indicate regions where perturbation theory is just no good (e.g. $x \to 0$ and $x \to 1$ in e-p).

2. Absence of obvious non-perturbative effects: The working hypothesis is that QCD describes hadrons. There are necessarily non-perturbative effects. Often these can be estimated crudely or inferred from experiment. Only if the perturbative prediction is larger can we hope to see it. It is often the case that the existence of important binding effects is signaled by an explicit breakdown of perturbation theory (as in #1).

3. The strong Kinoshita theorem: The infrared sensitivity of quark and gluon cross sections can be absorbed into hadron wave functions if Kinoshita's theorem is true for all differential cross sections in QCD. It has been tested in non-trivial cases [7] but not proven for QCD. The weak version, i.e. for total cross sections, has been proven [8].

4. A good approximation to the true solution: Pending verification, one assumes that non-perturbative effects will not drastically alter the perturbative predictions where those predictions appear to be self-consistent.

The QCD analyses of e-p and e^+-e^- rest on precisely these four foundations. The only simplification is that the weak Kinoshita theorem suffices for these specific applications. (From the present point of view, the operator product expansion plays the role of a special case of that theorem.) But all of the vagaries of the above framework are present in the most orthodox analyses. This is, after all, physics and not mathematics.

The role of the renormalization group is secondary. It is used here only to improve perturbation theory in situations where we compare scales that are exponentially different. But as such, it has played an important historical role in the conceptual development of QCD.

QCD is the only theory potentially capable of having both non-trivial bound states and perturbative behavior at high energies. An example of the importance of the perturbative nature of QCD is the Drell-Yan picture, which satisfies criteria #1 and 2 [4]. The reason that the process in fig. 1 dominates (when the quark distributions are properly defined to include the infrared-sensitive parts of all higher orders) is that it is lowest order in the quark-gluon coupling, g. For example, fig. 2, which also scales, is down by g^4 (or about 0.01 by present estimates) – as long as one stays away from the kinematic extremes of $M^2/S = 0$ or 1.

Any hadronic process, e.g. $p + p \to \pi + X$, can be addressed essentially by refining the parton model notions of distribution and decay functions and computing the parton amplitudes in QCD. Distribution functions have been discussed at length in the literature. Decay functions have also been shown to exist in field theory [9]. In general, they are totally unrelated to distribution functions. However, if their scaling behavior is governed by perturbation theory, then it is identical to the scaling behavior of distribution functions [10]. That is because they are both determined by the same radiative corrections: gluon bremsstrahlung and quark pair creation. The initial shapes of these functions do not satisfy criterion #1. Hence, they are determined by the non-perturbative aspects of QCD. So decay and distribution functions will satisfy the same differential equations in Q^2 but have different initial conditions.

Many features of hadronic reactions will be dominated by regions of the x's where perturbation theory is patently unreliable. We can, however, choose the kinematics to avoid these regions, as in inclusive, high transverse momentum experiments.

Logarithmic scaling violations from radiative corrections can be re-expressed in terms of effective, scale dependent distribution and decay functions. But these represent small corrections to the naivest parton picture (unless we consider enormous changes in scale). There is a class of phenomena which will be unmistakable evidence for QCD. These pertain to transverse momenta

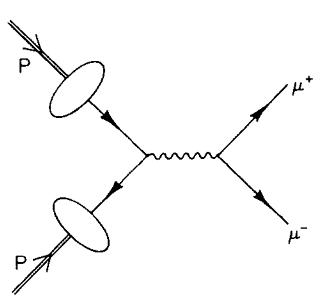

Fig. 1. The leading contribution to μ-pair production.

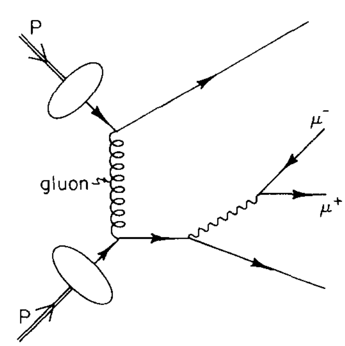

Fig. 2. Bremsstrahlung production of μ pairs in QCD, down by $g^4(M^2)$ from fig. 1 in cross section.

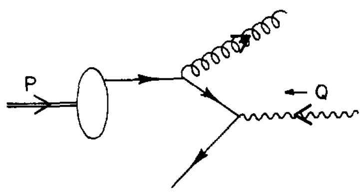

Fig. 3. Gluon bremsstrahlung in electroproduction as the origin of large transverse momenta.

within the effective distribution [11] and decay functions. Independent of perturbation theory, we know that the constituents of a bound state have transverse momenta on the order of the inverse size. In a renormalizable theory, radiative corrections as in fig. 3 give rise to transverse momenta that scale, i.e. grow proportionately to the energy of collision. At sufficiently high energies, these will dominate over the limited transverse momenta due to confinement. The exact formulas involve complicated convolutions over the longitudinal distribution functions [12], but, for protons, the following is a reasonable numerical approximation for $x \gtrsim 0.2$ to the mean initial quark transverse momentum [13]:

$$\langle p_T^2 \rangle \approx (300\ \mathrm{MeV})^2 + \frac{Q^2\,(1-x)}{8 \log Q^2/\Lambda^2}\,.$$

Λ^2 is determined by the size of g and is known to be roughly 0.25 GeV^2. The $(300\ \mathrm{MeV})^2$ is meant as a guess of the nonperturbative effects of confinement. x is the quark's longitudinal momentum fraction, and Q^2 is the momentum transfer in e-p or large angle scattering or the μ-pair mass in μ-pair production.

Radiative corrections to the "decay" of an outgoing quark into an observed hadron will also give the produced hadron transverse momentum relative to its parent quark, something like the above $\langle p_T^2 \rangle$. The exact calculation has not yet been done.

These transverse momenta can be observed directly in $\mu + \mathrm{p} \to \mu + \pi + \mathrm{X}$, $\mathrm{p} + \mathrm{p}(\bar{\mathrm{p}}) \to \mu^+\mu^-(\mathrm{W}) + \mathrm{X}$, and in hadron jet analyses. The same radiative corrections will also give rise to extra jets a small fraction of the time (e.g. 0.10 per extra jet.)

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