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AMBIGUITIES FROM QUARK MASSES IN THE RENORMALIZATION GROUP *

H. David POLITZER
California Institute of Technology, Pasadena, California 91125, USA

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Apparent discrepancies between calculations of the quark mass dependence of the lowest order β function in gauge theories are due to an inherent convention dependence of that function. Observable, physical quantities are, however, unambiguous.

Nachtmann and Wetzel [1] have demonstrated by explicit computation a phenomenon regarding the quark mass dependence of the renormalization group β-function. Specifically, the presence of masses can introduce a convention dependence into the β-function for the dimensionless gauge coupling even in lowest order. This phenomenon has not been widely appreciated; the original discussions of quark mass dependence [2] are ambiguous as to what conventions were used, and are consequently somewhat misleading.

The origin of the convention dependence can be seen if we try to reproduce the proof of the uniqueness of the lowest order β function, now in the presence of masses.

Let g and g' be two different renormalized versions of the same bare coupling constant, e.g., they differ as to which vertex is precisely specified, or with what momentum configuration or invariant decomposition they are defined. Renormalizability requires that

$$g' = g(1 + ag^2 + O(g^4)) . \tag{1}$$

By definition

$$\beta_{g'}(g') = M\, dg'/dM , \tag{2}$$

$$\beta_g(g) = M\, dg/dM , \tag{3}$$

where M is the scale of the renormalization point. So we conclude that

$$\beta_{g'} = M\frac{dg}{dM} + 3ag^2 M\frac{dg}{dM} + g^3 M\frac{da}{dM} + \dots . \tag{4}$$

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If no masses other than M enter into the definition of g, then by dimensional analysis $\mathrm{d}a/\mathrm{d}M = 0$, and the usual theorem holds: if $\beta_{g'}(g') = -b' g^3 + \ldots$ and $\beta_g(g) = -b\, g^3 + \ldots$, then $b' = b$. However, in the presence of non-zero quark m_i's in the vertex corrections included in g and g', a may well depend on m_i^2/M^2. Hence the β's are not unique even to $O(g^3)$.

The renormalization prescription which leads to the β-functions quoted in ref. [2] (though not explicitly stated there) is that the gauge coupling g be defined by a light (massless) quark-antiquark-gluon vertex. Any momentum configuration and any invariant decomposition for this vertex will lead to the same result to lowest order because, with each of these possible definitions, quark masses enter to lowest only *via* the quark loop in the gluon self-energy. (By the same token, using the ghost-gluon vertex to define g will lead to this same mass dependence of β.) The calculation is thus reduced to one identical to the lowest-order vacuum polarization in QED, and may be read off from any standard reference *. Using a three-gluon vertex to define g will imply a different form for the mass dependence of β.

Renormalizability ensures that any consistent prescription will lead to the same physical predictions, whether the β functions are the same or not. More precisely, any discrepancies between two calculations carried out to a given order must be yet higher order in the coupling constant. One may still ask, in the spirit of Moorhouse, Pennington and Ross [4], whether one particular prescription is better than others in the following practical sense: if we compute to lowest order and ignore yet higher orders, may one prescription be closer to the complete theory than another? That is to ask: can choice of a particular prescription minimize the numerical coefficient of g^2 in the next correction? Typically the answer is yes, but it is impossible to prove without actually computing that next correction. However, for the bulk of phenomenological applications, the use of the light quark-gluon vertex to define g seems a likely candidate because it is precisely that vertex which occurs in lowest-order amplitudes and is subsequently renormalized by higher orders.

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* See, for example, eq. (8.22) of ref. [3].