CHIRAL SYMMETRY, QUENCHED FERMIONS, NON-RELATIVISTIC QUARKS, η's AND GLUEBALLS ☆

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It is suggested that spontaneous chiral symmetry breaking, occurring prior to confinement, accounts for the successes of the non-relativistic quark model and quenched fermion Monte Carlo calculations. The validity and shortcomings of the latter are discussed, particularly with respect to glueballs and flavor-neutral mesons.

The spontaneous breakdown of chiral symmetry is a priori a conceptually distinct phenomenon from confinement. Kogut et al. [1] have reported evidence that these phenomena may also be dynamically distinct. Specifically, they find that the relevant phase transitions occur at different temperatures in lattice gauge theories. I propose that this be interpreted as evidence that the chiral breakdown occurs at shorter distances and weaker coupling than are relevant for confinement in the zero temperature theory. Hence, even massless quarks will have developed an effective, soft "mass" that affects their motion on the scale at which binding occurs. The observed strength of chiral breakdown, as reflected in $\langle 0|\bar{q}q|0\rangle$, is sufficiently large that the consequent m_{soft} of the quarks renders most hadron bound states essentially non-relativistic, hence accounting for the successes of non-relativistic quark models.

This substantial m_{soft} also serves as a justification of the quenched approximation or fermion loop expansion used in lattice Monte Carlo calculations [2], wherein the $\det|\not{D}|$ arising from quark loops is approximated by 1, its zero-loop value. Only "valence" quarks, created or annihilated directly by the operators whose expectations are evaluated, enter the calculation. These "valence" quarks pick up a substantial soft mass in the quenched approximation [2] and reproduce the non-relativistic picture for certain situations. Additional loop quarks would also develop soft masses. Their effects can be estimated using the analysis of Appelquist and Carazzone [3]. Typically, the effects of these additional quark pairs will be suppressed by $Q^2/m^2_{\text{soft}}(Q^2)$ for $Q < m_{\text{soft}}(Q)$, where Q is the available energy per quark. However, there is no such suppression in the effect of the additional pairs on renormalizations or in anything that has an incipient ultraviolet divergence. For questions involving $Q^2 \gg m^2_{\text{soft}}(Q^2)$, perturbation theory should suffice.

The validity of the non-relativistic picture depends on the question asked and the scale probed, and it provides expectations for the magnitude of corrections to the quenched approximation, where they are small and where large – as discussed below. An awkward but adequate definition of m_{soft} is given by the quark propagator

$$S_F(Q) = [\not{Q} - m_{\text{soft}}(Q^2)]^{-1}. \tag{1}$$

For large Q^2, in Landau gauge we know [4]

$$m_{\text{soft}}(Q^2) = g^2(Q^2)\langle 0|\bar{q}q|0\rangle_Q/3Q^2 + O(Q^{-4}) + O(m_{\text{bare}}). \tag{2}$$

The present conjecture is that this function becomes roughly 300 MeV on the scale relevant to confinement.

That chiral symmetry can break down in the quenched, zero-loop approximation is often a source of confusion. In fact, the existence of the requisite fermion condensate is a question of the ground state

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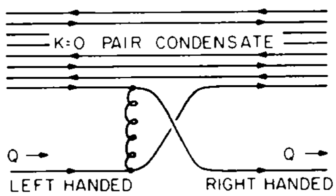

Fig. 1. Repeated left–right mixing through exchanges with the vacuum condensate generates an effective, soft quark mass, without the necessity of quark loop diagrams.

wave functional. This is determined self consistently by the functional integral method and need not be specified externally. "Valence" quarks acquire their soft mass from repeated exchanges as illustrated in fig. 1. The explicit gluon accounts for the g^2/Q^2 in eq. (2) for large Q.

For a vast variety of quantities the quenched approximation should be valid because the loop quarks will acquire an identical m_{soft}. Masses of many hadrons, such as P, ρ, and ψ, will receive only small corrections from the mixing with states with an additional massive quark pair. Strong decay widths, which only begin in the one-loop order, are a measure of this suppression. Since the relevant Q is the available kinetic energy and not the total hadron mass, I suggest $\Gamma_\omega/\Gamma_\rho \sim 1/15$ as an estimate of the effect of an extra pair.

Other static quantities besides masses, such as magnetic moments, g_A's, and matrix elements of local operators which give structure functions (including higher twists) should also be calculable in a quenched approximation. Rather, it is the ratios of these quantities that is reliable. The absolute magnitude relative to the short distance coupling involves sizable renormalization corrections from loops. This correction can be estimated as follows. Imagine computing a physical mass m on a lattice with fixed spacing a and fixed bare coupling $g_0(a)$ in two ways: first with no loops and second including one-loop effects. Aside from O(1/15) mixing corrections, there are also different charge renormalizations (see fig. 2). Hence

$$\frac{m_{0\text{-loop}}}{m_{1\text{-loop}}} \approx \frac{\exp(-2b_0 g_0^2)^{-1}}{\exp(-2b_1 g_0^2)^{-1}}, \tag{3}$$

where $b_{0,1}$ are the 1-loop β-function coefficients without and with fermions. For the a of existing Monte Carlo calculations, u, d, and (only marginally) s quarks are relevant. With a typical $g_0 = 1$, the ratio in eq. (3) is 3.0 ± 0.3. Roughly the same factor should apply to any mass. Hence, the Λ deduced from a zero loop mass calculation will be only one third the size of the true Λ, which characterizes the short distance coupling.

This discussion allows us to interpret the glueball masses computed in pure SU(3) glue theory. Aside from O(1/15) corrections and mixing with $q\bar{q}$ states (discussed below), this calculation should be reliable either if expressed as a ratio to another calculated mass (e.g., the proton's) or if made smaller by the inverse of the factor in eq. (3) when expressed as a multiple of some Λ.

The effect of extra loops in calculating the mass of the η is not suppressed by m_{soft} because the axial anomaly is quark-mass independent. However, by identifying the dominant effect of quark loops, we can organize a practicable quenched Monte Carlo calculation. Consider first a one flavor theory. The would-be Goldstone boson mass, m_0, vanishing in the chiral limit, can be computed from the $T \to \infty$ exponential fall-off of the two-point function

$$\int d^3x \, \langle \bar{q}\gamma_5 q \, \bar{q}\gamma_5 q \rangle = A^2 \exp(-m_0 T), \tag{4}$$

as indicated in fig. 3a. The correlation indicated in fig. 3b can be computed in the quenched approximation equally well. It should yield

$$\int d^3x \, \langle \bar{q}\gamma_5 q \, \bar{q}\gamma_5 q \rangle = -A^2 \int_0^T dt \exp(-m_0 t) K \exp[-m_0(T-t)]$$

$$= -A^2 KT \exp(-m_0 T), \tag{5}$$

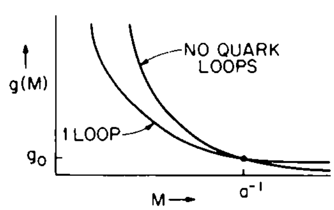

Fig. 2. Renormalization of the gauge coupling, with and without fermion loops.

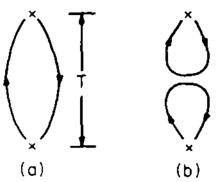

Fig. 3. Two contributions to the q$\bar{q}$-meson propagator. The quenched calculation includes all possible gluons but only those quarks explicitly indicated.

because as $T \to \infty$ the dominant configurations have intermediate states of mass m_0; the pure glue configuration is effectively localized at time t (fig. 4a). If we could evaluate the same correlation with yet more quark loops, the dominant configurations would again be intermediate states of mass m_0 connected by local factors of $-K$ at arbitrary times. The sum of terms indicated in fig. 3b exponentiates to give a factor of $\exp[-(m_0 + K)T]$. So $m_0 + K$ is a reasonable first estimate for the meson mass, calculable with quenched fermions. However, a few comments are in order:

(1) $(m_0 + K)^2$ is the meson self-energy at $Q_\mu = (m_0, \mathbf{0})$ and not on shell. (The extrapolation of this self-energy is discussed below.)

(2) A Monte Carlo calculation involving a finite number of loops will not improve this estimate because it will only reproduce a term in the series that exponentiated.

(3) The physical mass will also be altered by mixing with the nearest appropriate glueball. However, this effect can be estimated by a quenched Monte Carlo. Just as fig. 5a gives a pure glueball mass m_G from

$$\int d^3x\, \langle GG\rangle = B^2 \exp(-m_G T), \tag{6}$$

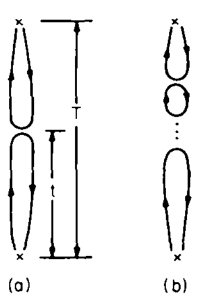

Fig. 4. $T \to \infty$ behavior is dominated by q$\bar{q}$ configurations separated by (a) a local interaction at t or (b) several local interactions.

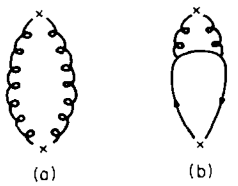

Fig. 5. The glueball propagator and glueball – q$\bar{q}$-meson mixing, with quarks indicated explicitly but all possible gluon interactions implied.

where G is a glueball interpolating field, fig. 5b gives the mixing parameter C

$$\int d^3x\, \langle G\, \overline{\bar{q}\gamma_5 q}\rangle = ABC \exp(-m_0 T). \tag{7}$$

Note that the large T behavior will again be dominated by the mass m_0 state.

The general issue of mixing can be addressed using the self-energy matrix $\pi_{ij}(Q^2)$ where for suitable interpolating fields ϕ_i

$$\int d^4x\, \exp(iQx)\langle\phi_i(x)\phi_j(0)\rangle = [Q^2\delta_{ij} + \pi_{ij}(Q^2)]^{-1}. \tag{8}$$

In principle, one should diagonalize π_{ij} as a function of Q^2 and identify the poles of eq. (8). If π_{ij} were constant and real, there would be two simple poles for the η_0 meson and glueball system. In practice, we only have estimates for the values of π_{ij} at various points and must make the standard assumptions of the weakness of cuts (i.e., narrowness of widths) and of the distance to other poles. This amounts to assuming π_{ij} to be slowly varying over the range of mixing.

Eq. (7) implies that the off-diagonal η_0-glueball element is

$$\pi_{\eta G}(m_0^2) = C(m_G^2 - m_0^2). \tag{9}$$

A serious possible criticism is that the extrapolation of π_{ij} from m_0^2 to the range of $(m_0 + K)^2$ to m_G^2 may be so large as to be suspect for $m_0 \to 0$.

The generalization to several flavors is simple. For massless u and d and a light s-quark, the neutral mesons π^0, η, η', and the physical glueball are the eigenvectors of

$$\pi_{ij} = \begin{pmatrix} K^2 & K^2 & K^2 & Cm_G^2 \\ K^2 & K^2 & K^2 & Cm_G^2 \\ K^2 & K^2 & (\sqrt{2}m_K + K)^2 & Cm_G^2 \\ Cm_G^2 & Cm_G^2 & Cm_G^2 & m_G^2 \end{pmatrix}, \quad (10)$$

in the $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, G basis.

The effective non-relativistic quark interpretation suggested above also has implications for the expected widths of the unmixed, pure glueballs. They should be narrow but for a reason totally unrelated to the narrowness of the ϕ, ψ and Υ. The decay of these latter mesons is imagined to be a short distance process requiring the annihilation of a heavy quark pair [5]. The subsequent production of light quarks does not substantially alter the decay amplitude, (and these quarks are effectively massless because of the large available Q). In contrast, glueball decay does not require that anything annihilate. Rather, it is a long-distance process like typical strong decays. It occurs via the mixing with states with additional (now softly massive) quarks. Glueballs that are allowed to decay into two pions are therefore expected to have unmixed widths comparable to the ω, i.e., 10 MeV, which also needs two extra pairs to decay.

Physical glueballs may be much broader due to the mass-mixing with neutral $q\bar{q}$ states, the width depending on the mixing angle and the decays of the mesons with which they mix. All $q\bar{q}$-glueball mixings can be estimated from quenched fermion Monte Carlo calculations as indicated by fig. 5b.

Hamber and Parisi [6] have recently estimated a parameter closely related to K of eq. (5) from the connected part of

$$\int d^4x_1\, d^4x_2 \langle \bar{q}\gamma_5 q(x_1)\, \bar{q}\gamma_5 q(x_2)\rangle,$$

which gives the shift in π_{ij} at $Q^\mu = 0$. They find $\delta\pi_{ij}(0) = 200 \pm 100$ MeV. However, their proposed justification for the calculation is somewhat different relying instead on an expansion in positive powers of n_f, the number of flavors.

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