THE STRUCTURE OF HADRONS IN QCD

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We show that in QCD the instanton gas is unstable under collapse into a dense strong coupling phase. However, in the presence of color fields due to quarks, a dilute gas can coexist with the dense phase. This leads to a derivation, directly from QCD of a confining bag model of hadrons.

In previous work [1] we have studied the effects of semi-classical vacuum fluctuations-instantons [2] and merons in QCD. Whereas at very short distances asymptotic freedom ensures that perturbation theory is applicable, the non-perturbative instanton induced effects rapidly dominate at scale sizes where the coupling is still sufficiently small $[g^2/8\pi^2 \sim \frac{1}{25}]$ that semi-classical methods are valid. These effects can be quite important; yielding, as we have shown, qualitatively new heavy quark interactions [1,3] and a mechanism for spontaneous chiral symmetry breaking [1].

Our previous efforts gave only a rough notion of these effects, limited as they were to a dilute-gas-approximation where instanton interactions were all but neglected. Furthermore, we could see no way of suppressing the large scale (and therefore, due to "infrared slavery", strongly coupled) fluctuations of the vacuum. In this letter we report on our advances toward the solution of these problems [4].

We find that the euclidean vacuum can be regarded as a four-dimensional ensemble of permanent color magnetic dipoles (instantons or meron pairs) with a paramagnetic permeability, $\mu > 1$, that can be calculated by standard methods. In the presence of an external color field (due to quarks), large scale fluctuations are suppressed and we can take full account of the instanton interactions. However, below a critical field strength, E_c, this phase is unstable and a first-order phase transition occurs to a dense phase consisting of closely packed instantons and merons, and possibly other things, where we believe $\mu = \infty$. This leads to a simple bag-like picture of hadrons as consisting of quarks confined to a region of space-time, which is in a very dilute (abnormal) vacuum phase, in equilibrium with the dense vacuum (normal) phase outside the bag. The quarks are confined to the region of dilute phase where their dynamics is simple and, as it turns out, they are also shielded from the large scale vacuum fluctuations outside the bag. This produces a picture of hadronic structure similar to the MIT bag model [5], although there are many important differences, not the least being that it is derivable directly from QCD. This letter is not meant to be complete but rather a summary of the detailed treatment of ref. [4], which see for details.

Let us first consider pure QCD with heavy quarks only. In a previous paper [1], we have determined that the long range instanton-anti-instanton interaction is that of four dimensional magnetic dipoles. This corre-

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spondence can be greatly extended, and we find that the instanton-anti-instanton interaction is dipolar down to separations which are of order twice the scale size, ρ, of the instantons. Thus, for moderate densities instantons are simply permanent magnetic dipoles, with dipole moment $D_{\mu\nu}^{\alpha} = \rho^2 \eta_{a\mu\nu}$. Such four dimensional dipoles can be either self dual (instantons) or anti-self dual (anti-instantons). They produce a field, $F_{\mu\nu}^{\alpha} = T_{\mu\lambda}(\hat{x}) D_{\lambda\epsilon}^{\alpha} T_{\epsilon\nu}(\hat{x})/x^4$, (where $T_{\mu\nu}(\hat{x}) = \delta\mu\nu - 2\hat{x}_\mu \hat{x}_\nu$) of opposite duality. Their interaction energy with an external field $F_{\mu\nu}^{\alpha}$ is simply $(2\pi^2/g^2) F_{\mu\nu}^{\alpha} D_{\mu\nu}^{\alpha}$. Thus, instantons (self dual dipoles) interact only with anti-instantons (anti-self dual dipoles).

Such a medium of permanent dipoles is paramagnetic, and the permeability of the medium, μ, is greater than one. In field theoretic language this corresponds to anti-screening of color or asymptotic freedom, namely the effective static (small momentum) coupling, g^2, is increased $g^2 \to g^2\mu$. In ref. [1], we evaluated μ in a dilute gas approximation, and showed how μ appeared as the coupling constant renormalization for the long range heavy quark potential. By adapting standard techniques for dealing with polar media [6], we can now calculate the permeability even when it is very large, as it is in the non-dilute instanton gas. We show [4] that

$$\mu = \eta + \sqrt{1+\eta^2}, \quad \eta = \frac{\pi^2}{2} \int \frac{d\rho}{\rho} x(\rho)\, n(\rho)\, \rho^4 \qquad (1)$$

where $x(\rho) = 8\pi^2/g^2(\rho)$ is the effective coupling ($\sim -\ln(\overline{\rho\mu})$) and $n(\rho)$ the instanton density. When η is small and n equals the dilute gas density $n_0(\rho)$, $\rho^4 n_0(\rho) = 0.1\, x^6(\rho) \exp[-x(\rho)]$, this coincides with the result of ref. [1].

The above expression leads to a very large permeability, $\mu \sim 20$, even when we only include instantons up to an integrated density, f, of one (roughly speaking, $\mu \sim 2fx_c$ where $\rho_c\bar{\mu} = \exp(-x_c/11)$ is the cutoff on instanton scale size). Meron pairs can be treated in an analogous fashion (they are again dipoles) increasing μ, and we have given arguments [1] that for scale sizes where $x \sim \frac{32}{3} + 6.55 \approx 17.2$, these pairs ionize leading to a divergent μ. Although this phase is not understood in detail (free merons are inherently non-abelian) it is already clear that there is a calculable $\mu \gtrsim 20$ in the vacuum, and the precise manner in which it diverges may not be too important (except as a matter of principle in proving absolute confinement).

Since they are attractive, the interactions increase the density of instantons. To compute this effect one minimizes the free energy $F = F_0 + F_{\text{int.}}$, with respect to the density of instantons of size ρ, where F_0 is the free energy of the perfect gas and $F_{\text{int.}}$ contains the interaction energy. We calculate the latter by mean field theory, considering a given instanton to be in a spherical cavity of radius R and interacting with a polar medium of permeability μ. The result is [4]

$$\frac{\partial F}{\partial n(\rho)} = 2 \log \frac{n(p)}{n_0(p)} + 12\, x(\rho) \frac{\mu - 1}{\mu + 1} \left(\frac{\rho}{R}\right)^4 . \qquad (2)$$

To second order in the density this agrees precisely with the virial expansion of F, where R is a hard core cutoff on the dipolar potential. The optimal value of R is determined to be $\sim 2.2\rho$ [4]. The density is then determined by $\partial F/\partial n(\rho) = 0$, where μ is calculated via eq. (1).

This dipole gas is unstable under a collapse to densely packed instantons, which then dissociate into merons. This is the (normal) vacuum phase which cannot be treated without (at least) a fuller understanding of free merons and in which μ probably diverges (at least $\mu \gtrsim 20$). However, we have discovered that in the presence of color fields (due, say, to quarks) the dilute phase will be stable. This is due to the phenomenon of magnetostriction which causes the density of instantons to *decrease* in the presence of quark fields. This is the analog of ordinary electrostriction in dielectrics except that in our case $\epsilon = 1/\mu < 1$. Furthermore, at a critical value of the field strength, E_c, the dilute and dense phase can be in equilibrium at a planar boundary (see fig. 1).

To see this we consider instantons in the presence of an external constant color field. For a static configuration we can take an electric field, $E_i^{\alpha} = \delta_{\alpha 3}\delta_{i3}E$.

The permeability due to instantons μ, is equivalent to a dielectric constant $\epsilon = 1/\mu$ and for fields of the strength we shall consider the Yang–Mills equations in the instanton medium can be treated as linear [4]. The appropriate thermodynamic potential is now [7] $\widetilde{F}(n, E) = F_0 + F_{\text{int.}} - \frac{1}{2}\bar{E}\cdot\bar{D}$ where $D = E/\mu$, and the resulting density of instantons is given by

$$n(\rho) = n_0(\rho) \exp\left[6x(\rho)\frac{\mu-1}{\mu+1}\left(\frac{\rho}{R}\right)^4 - \frac{\pi^2 E^2 \rho^4}{4(1+\mu^2)} x(\rho)\right]. \qquad (3)$$

Note that the dependence of $n(\rho)$ on E implies a sub-

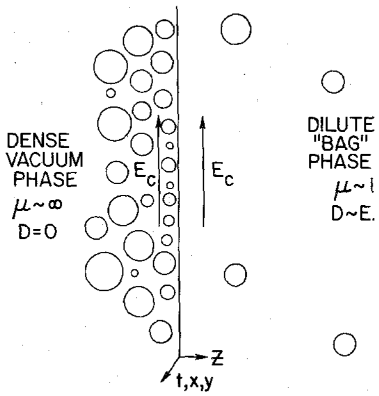

Fig. 1. The boundary conditions at a planar boundary separating the dense vacuum phase from the dilute "bag" phase. At the boundary the tangential component of $\boldsymbol{E}$ and the normal component of $\boldsymbol{D}$ are continuous.

stantial reduction in the density in high field regions, and the larger the scale size of the instanton (ρ) the larger the reduction factor (this is because the dipole moment $\sim\rho^2$). Thus the field acts like an infrared cutoff, suppressing large instantons, and for large E the gas will be very dilute and completely calculable.

Eqs. (3) and (1) can be easily solved. We chose $R = 2.2\rho$ and calculate $n(\rho)$, μ, and D as a function of E [4]. The result for $D(E)$ is plotted in fig. 2 and values of μ, E, x_p (the peak value of x in the η integral) and f (the fraction of space occupied by instantons) are listed in table 1 for various points, 1 ... 6, along the curve. We note the following features:

(1) For all the points 1 through 5, our approximations are very good, since the density of instantons is not so big as to invalidate the dipole approximation which leads to eq. (2), and the external field is not so big as to cause saturation of the induced magnetisation [4]. This is especially true in the dilute phase and at the point of instability (3). Furthermore, the existence of the instability (and the value of μ and E at this point) is remarkably insensitive to changes in the form of the approximations [4].

(2) There is clearly an instability, starting at point 3, where the instanton gas is unstable. This is analogous to the instability of the Van der Waals gas and usually signals the existence of a first-order phase transition to a condensed phase. At point 5 our treatment breaks down, since the instantons are beginning to be dense and meron-pairs to dissociate. Here we believe that μ

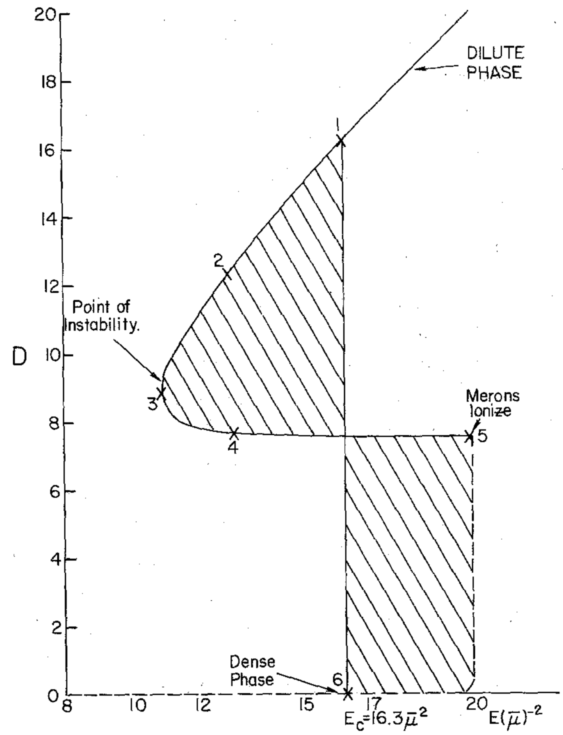

Fig. 2. The phase diagram for the instanton gas. The solid line can be reliably calculated using the methods described in the text.

will rapidly increase (and probably diverge, leading to the dotted line in fig. 2). In that case, one can determine the critical field at which the phase transition occurs, $E_c \approx 16.3$, by the standard Maxwell construction. For the two phases to be in equilibrium with the same value of E (which is the appropriate condition since the Yang–Mills equations imply the continuity of the component of E parallel to the phase boundary) the

Table 1
The values of the permeability (μ), the field strength (E), the peaked x in the η integral (x_p) and the fraction of space (f) occupied by instantons for representative points on the phase diagram.

	μ	$E/\bar{\mu}^2$	x_p	f
1	1.016	16.3	22.6	0.0014
2	1.051	12.9	20.7	0.005
3	1.17	10.85	18.7	0.02
4	1.67	12.94	17.7	0.06
5	2.7	20	16.8	0.15
6	∞	E_c = 16.3	?	?

value of $\widetilde{F}$ must be the same for both phases [7]. Since $d\widetilde{F} = -D\,dE$, this requires that $\int_{E_c}^{E_c} D(E)\,dE = 0$. The difference in the free energies of the dilute and dense phase is then given by $\frac{1}{2}E_c^2((1/\mu_1) - (1/\mu_2))$, where $\mu_1(\mu_2)$ is the permeability in the dilute (dense) phase. This will turn out to be the "bag constant", B, and since $\mu_1 \approx 1$, $\mu_2 \approx \infty$, we have that

$$B \approx (1/2\mu_1)E_c^2 \simeq 130\bar{\mu}^4 \tag{4}$$

is the energy density that it costs to create the abnormal (dilute) phase.

In the absence of quantitative control over the dense phase we can only trust this estimate of B to within perhaps a factor of two. *What is certain, however, is that there exist two such phases – one dilute with small coupling ($x \sim \frac{1}{23}$) and one dense in which $\mu \sim \infty$ (or at least $\gtrsim 20$).* This, we shall argue, leads to confinement of quarks (or at least partial confinement) and to a bag-like picture of hadrons. Furthermore, since the quarks will be confined to the dilute phase, the structure of hadrons will presumably depend little on the details of the dense vacuum phase. We can therefore understand how in a theory where the forces are so strong that they prevent one from pulling a quark out of a hadron the simple quark model of hadrons can be so successful.

Let us first consider heavy quarks. If these are inserted into the vacuum the color fields produced by the quarks will expel instantons (magnetostriction), creating a bag or flux tube (which of course has infinite extent in the time-like direction) in which the dilute phase exists. The shape of the bag will be determined by solving the Yang–Mills equations (with $x \sim \frac{1}{23}$) inside the bag with the boundary conditions, $\hat{n}\cdot \boldsymbol{E} = 0$ and $|\boldsymbol{E}| = E_c \equiv \sqrt{2B}$ on the surface. These are, of course, the standard MIT bag equations. In the limit of a very widely separated quark-antiquark pair, the tube will be (in the three spatial dimensions) a cylinder of diameter $d = (256/3BX)^{1/4}$ and the interaction energy will increase linearly with separation with an energy density per unit length $\epsilon = \sqrt{(64\pi^2 B/3x)}$ [‡1].

[‡1] All dimensional parameters are expressed in terms of $\bar{\mu}$, the renormalization scale parameter. Estimates of the Regge slope or the proton radius lead to $\bar{\mu} \sim 70$ MeV. However, this depends on how we precisely define the coupling. This and other process dependent ambiguities make it difficult at this moment to compare $\bar{\mu}$ (used here) with the scale parameter that enters into deviations from asymptotic freedom.

This bag shields the quarks inside it from large color field fluctuations in the vacuum. This is the QCD analog of the Meissner effect. Since the penetration of a field into a QCD bag is governed by the ratio $(\mu_{in}/\mu_{out})(1/\mu_{vac.}) \approx 0$, which is the same as $\mu_{in}/\mu_{out} = \frac{0}{1} = 0$ that occurs at the surface of a superconductor from the point of view of a vacuum fluctuation the QCD bag looks like a cylinder of color superconductor which expels the color field [4].

Turning now to light quark states we must confront the question of quark propagation in the instanton gas and chiral symmetry breaking. The light quarks have very small mass parameters (the value of m in the lagrangian) but due to spontaneous chiral symmetry breaking achieve large dynamical masses (the term $m(p)$ in the fermion propagator $[\not{p} + m(p)]^{-1}$). In ref. [1] we showed that the effective four fermion interactions generated by instantons (even in a moderately dilute phase) is sifficient to render the SU(2) × SU(2) symmetric vacuum unstable, and to produce a Goldstone vacuum with a nonvanishing fermion mass and a bound state Goldstone boson.

In the dilute gas approximation, $m(p)$ could be calculated via a Hartree–Fock equation. Our investigation of this equation, as well as our calculation of $m(p)$ in the case of one massless flavour leads us to conclude that once the density of instantons (or meron pairs) is substantial, $m(p)$ will be very large at small p. Thus, if a bag exists as discussed above, the inside of the bag will be, since the density of instantons is so low, chirally symmetric and the quark masses will be given by their bare values. Outside the bag, however, the density is so high, that the vacuum will exhibit spontaneous chiral symmetry breaking, generating a *very large* quark mass (for small momentum). Thus our bag acts also as a mass bag, which confines the quarks to a region where they are light.

If we imagine that the phase boundary (the bag surface) responds slowly to the fluctuating fields created by the quarks inside the bag, we can determine, as before, the conditions for phase equilibrium. Given a collection of quarks in a singlet state we again must solve the Yang–Mills and Dirac equations (with $x \sim \frac{1}{23}$) for the quarks and gluons inside the bag (treated as static) with the boundary conditions $\hat{n}^\mu F_{\mu\nu}^{\alpha} = 0$ on the surface ($\hat{n}_\mu$ is the spacelike normal to the bag surface). The boundary condition for the quark field is determined by the discontinuity of the quark mass at boundary. To first approximation, $m_q \sim \infty$ outside the bag, leading to $i\not{n}\psi = \psi$ on the surface. This coincides with the MIT boundary condition for the quark field, and appears to introduce explicit chiral symmetry breaking. However, we must now recall the QCD vacuum phase outside the bag is a Goldstone vacuum with the zero mass (if the quark masses vanish) Goldstone bosons. These will necessarily couple to the quarks at the surface of the bag restoring chiral symmetry.

To account for the Goldstone boson excitations of the dense phase one can introduce a non-linear chiral lagrangian outside the bag, $\mathcal{L} = \frac{1}{2}(D_\pi \mathbf{\Pi})^2$ (in the case of SU(2) and with $D_\Pi = 1/(1 + (\pi^2/f_\pi^2))\,\partial$ to describe the self-interacting Goldstone boson in the limit of infinite dynamical quark mass). The effective pion field (which is really $\bar{\psi}\gamma_5(\boldsymbol{\tau}/2)\psi$) satisfies $D_\pi^2 \mathbf{\Pi} = 0$ outside the bag and on the surface couples to the pseudo-scalar density.

$$n^\mu \bar{\psi}\gamma_\mu\gamma_5(\boldsymbol{\tau}/2)\psi = \mathrm{i}\,\tfrac{1}{2}\,\bar{\psi}\gamma_5\boldsymbol{\tau}\,\psi = f_\pi n^\mu D_\mu^\pi \mathbf{\Pi}, \qquad (5)$$

which ensures the conservation of the axial current A_μ, $\bar{\psi}\gamma_\mu\gamma_5(\boldsymbol{\tau}/2)\psi$ in the bag and $f_\pi D_\mu \mathbf{\Pi}$ outside.

Finally we must balance the pressures of the two phases at the boundary, where now in addition to the chromodynamic pressure of the color fields in the bag we have the kinetic pressure of the quarks helping to balance the vacuum pressure of the dense phase plus the chiral pressure of the pion field outside the bag. This results in

$$B \cong \tfrac{1}{2}E_c^2 = \tfrac{1}{2}n^\mu D_\mu(\bar{\Psi}\Psi) - \tfrac{1}{2}(D_\pi \mathbf{\Pi})^2 - \tfrac{1}{4}\mathrm{Tr}(F_{\mu\nu\alpha}^{\ \ \alpha} F_{\alpha}^{\mu\nu}) \qquad (6)$$

at the surface. These equations are similar to the MIT bag model, differing only in the inclusion of the Goldstone mode present in the vacuum outside the bag. The treatment of the phase boundary as static must be, of course, justified or superceded by an improved treatment.

Even at this level of approximation there will be differences between our model and the MIT bag, due to the coupling of "bag states" with the Goldstone pion. This will have an effect on the hadronic spectrum, and will also allow for a simple calculation of pion couplings. The pion itself, of course, is not, strictly speaking, a bag state but rather a Goldstone excitation of the dense vacuum, and its properties (f_π, for example) are determined by this phase. If one considers, however, $I^G(J^P) = 1^-(0^-)$ states inside the bag one can see how the bag tends to collapse to a zero mass bound state. Following Horn and Yankelowitz [8] one includes in the energy balance the attractive (in the Π channel) R^{-3} force due to the determinental interaction induced by instantons. We find, with the density of instantons that exists in the bag, that this interaction causes the bag to contract, and the mass to decrease, to a point where the mass vanishes and the bag no longer exists [4]. An interesting open question is whether there exists another metastable and heavy baglike pion.

There are other important differences between our bag and the MIT model; in particular, there are surface effects. Much as in the case of a liquid–gas phase boundary the surface is (in a five-dimensional sense) the time average of a surface which is evaporating and absorbing instantons in equilibrium. The instantons inside the bag are attracted to the surface and will be denser near it – thus an effective surface layer will be formed. We have evaluated the density (and μ) near the surface and find that it drops rapidly to its small gaseous value at a distance $\Delta \sim 0.2\rho_c$ from the surface, where $\rho_c = 0.12(1/\mu)$ is the (peaked) size of the instantons inside the bag [4]. Since the radius of the bag (for say the nucleon) turns out to be of order $0.25(1/\mu)$, this means that we still have a rather sharply defined bag. However, there will be important surface effects such as surface energy and tension, which will change the pressure equation. Also, in our picture the energy shift due to zero point oscillations in the bag comes from the instanton determinants (with interactions taken into account) and is finite and in principle calculable.

Finally, we do have instantons inside the bag, and especially within the surface layer, and there will induce non-perturbative interactions between quarks, which are likely to dominate over perturbative gluon interactions and must be included in calculations of hadronic masses, couplings, and interactions.

In conclusion we have established the existence of two vacuum phases in QCD, a dense phase consisting, presumably, of ionized merons in which the (coupling) permeability is infinite (or at least very large) and a dilute phase to which quarks are confined; leading to a bag-like picture of hadrons. There remains the difficult theoretical problem of gaining control over the dense vacuum phase. An easier task will be to work out the phenomenological implications of our chirally symmetric bag, with surface effects and instanton induced interactions.

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