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STRING-LIKE CONFIGURATIONS IN THE WEINBERG-SALAM THEORY *

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The Weinberg-Salam theory of electromagnetic and weak interactions admits classical configurations in which a pair of magnetic monopoles is bound by a flux string of the Z^0 field. They give rise to Regge trajectories of excitations with a mass scale in the TeV range.

1. Introduction

Much of the interest that has been aroused recently in the monopoles, strings and other special solutions of gauge field equations seems to be directed at the general existence and structure of solutions in arbitrary model systems, rather than at the nature of solutions in specific models which are more relevant to the actual physical problems. This is understandable because, first of all, the very possibility and beauty of such solutions is so fascinating, with implications which probably will extend beyond our current concerns, and also because there are still a great deal of uncertainties in pinning down the right theories.

The existence of string-like objects in an Abelian Higgs model has been pointed out by Nielsen and Olesen [1] as a simplified model for hadronic strings. This analogy can be carried a bit closer by attaching monopoles to the ends of a string [2], but the problem of constructing non-Abelian strings, especially those representing baryons, has not been satisfactorily resolved yet. In other words, it is not yet clear whether the Nielsen-Olesen type picture is the correct picture for hadronic strings.

As for monopoles, 't Hooft [3] and Polyakov [4] have shown the existence of finite-energy solutions in an SO(3) Higgs model. However, the more relevant model for electromagnetic and weak interactions is the SU(2) × U(1) model of Weinberg [5] and Salam [6]. We would like to assert in this paper that the Weinberg-Salam theory admits classical solutions which look very much like the Nielsen-Olesen hadronic string with a pair of monopoles attached to its ends. The energy scales are

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vastly different, however. Both the mass of the monopole and the tension of the string are estimated to be in the TeV range.

The qualitative arguments for the existence of such solutions are as follows. Consider a monopole solution of the Wu-Yang type [7], in which the isospin direction is locked with the radial vector. A Higgs field is necessary to smooth out the singularity at the origin and make the energy finite. In the model of 't Hooft and Polyakov the Higgs field is an isovector

$$\phi^i(r) = f(r)x^i/r\,, \tag{1}$$

where $f(r) \sim$ const asymptotically, and $f(0) = 0$ so that ϕ^i is well defined everywhere.

In an SU(2) × U(1) model where the Higgs field is an isospinor ϕ^α ($\alpha = 1,2$), the corresponding solution would be (in polar coordinates)

$$\phi \sim \begin{pmatrix} \cos \frac{1}{2}\theta \\ \sin \frac{1}{2}\varphi\ \mathrm{e}^{i\varphi} \end{pmatrix}, \tag{2}$$

for $r \neq 0$, but this still leaves an ill-defined phase along the negative z axis ($\theta = \pi$). Therefore ϕ would have to vanish for $\theta = \pi$. That means that one cannot have a simple monopole, but it must be accompanied by a string. Suppose now we go far away from the origin along the negative z direction: Since there is a U(1) gauge field, and the effect of the monopole may be ignored, ϕ is essentially of the form

$$\phi = \begin{pmatrix} 0 \\ f(\rho)\ \mathrm{e}^{i\varphi} \end{pmatrix}, \qquad \rho = \sqrt{x^2 + y^2}\,. \tag{3}$$

Thus one is dealing with a semi-infinite Nielsen-Olesen string made up of a flux of a combination of gauge fields. Such a system is unstable, but we can obtain a stable system if we terminate the string by putting another monopole of opposite sign and spin the system. The emerging picture is exactly that of the string model of mesons.

The long-range fields associated with the system behave as follows. Far away from the system, we have a linear combination (the real electromagnetic field) of U(1) and SU(2) gauge fields created by the pair of magnetic poles. Along the string, however, the two components behave differently. The U(1) part has a return flux through the string, whereas the SU(2) part does not. Thus the poles are genuine SU(2) monopoles. Because of this situation we expect such a system to be essentially stable if the two poles are kept sufficiently far apart; the poles will maintain their integrity since each has a topological quantum number.

The rest of the paper is devoted to the mathematical details.

2. General formalism for dealing with monopoles and strings

In this section we review a mathematical formalism developed elsewhere [8] *, which makes transparent the handling of classical solutions of a gauge field in the presence of a non-vanishing Higgs field. We denote the latter by ϕ^α, $(\alpha = 1,2, ...)$ or simply ϕ; it corresponds to a minimum of the total energy of the system. Usually this means that both the potential and kinetic energies are separately minimal:

$$V(\phi) = \min\,, \qquad D_\mu \phi = 0\,, \tag{4}$$

where D_μ denotes a covariant derivative. These two conditions are compatible with each other since the second equation states that the ϕ's at two neighboring points are related by an infinitesimal action of a gauge group, under which $V(\phi)$ is invariant. Thus an arbitrary path joining two points in space is mapped into a path in the space of ϕ's with $V(\phi) = \min$. In particular, a closed loop must be mapped into a closed loop since ϕ must be one-valued. This puts a stringent condition on the admissible distribution of the gauge fields. In fact from eq. (4) follows

$$[D_\mu, D_\nu]\phi \propto F_{\mu\nu}\phi = 0 \qquad \text{for all } (\mu\nu)\,, \tag{5}$$

where $F_{\mu\nu}$ acts on ϕ as a transformation matrix. Eq. (5) means det $F_{\mu\nu} = 0$ for all $(\mu\nu)$, since by assumption $\phi \neq 0$. Thus the gauge field can have a non-vanishing component in a medium (with $\phi \neq 0$) only if the corresponding generator of the gauge group has a zero eigenvalue in the representation ϕ. One may call this the statement of a generalized Meissner effect. In the real Meissner effect, the group is U(1), ϕ is a complex one-component field, and hence $F_{\mu\nu} = 0$. A similar situation holds for an SU(2) gauge field with an isospinor ϕ,: all the components $F_{\mu\nu}{}^i$, $i = 1,2,3$, must vanish because no generators have zero eigenvalues on ϕ.

On the other hand, an isovector ϕ allows one of the generators to vanish, so that the corresponding component of $F_{\mu\nu}{}^i$ survives the Meissner effect. This phenomenon also occurs in general if there are more than one commuting generator (i.e., if the group is not simple, or if its rank is $\geqslant 2$), because we can make a linear combination of such commuting generators annihilate a ϕ.

The above conditions, however, are not sufficient in general for satisfying eqs. (4) and (5) everywhere. There may be points (monopoles) or lines (strings) on which the gauge potential is singular and the distribution of ϕ ill defined. So the true solution must necessarily make ϕ vanish at the singularities and violate these equations in their neighborhood. But the topological properties of the mapping $x \to \phi(x)$ around the singularities are not affected by such deviations.

As was said above, eq. (4) or (5) puts a severe restriction on the gauge potentials

* The line of reasoning that follows is often found implicitly stated in the literature, but it has not been properly emphasized in our opinion. See in particular, ref. [9].

$A_\mu{}^i$. In fact, we can solve eq. (5) for A_μ. For example, for the case of isovector ϕ and A_μ, eq. (4) reads

$$\partial_\mu \boldsymbol{\phi} - g\boldsymbol{A}_\mu \times \boldsymbol{\phi} = 0 \tag{6}$$

from which follows first

$$\boldsymbol{\phi} \cdot \boldsymbol{\phi} = \text{const} , \tag{7}$$

so we will conveniently choose $\boldsymbol{\phi}$ to be a unit vector. Then the solution to eq. (5) is [8,9]

$$g\boldsymbol{A}_\mu = \boldsymbol{\phi} \times \partial_\mu \boldsymbol{\phi} + a_\mu \boldsymbol{\phi} , \tag{8}$$

where a_μ is arbitrary, and

$$\begin{aligned} \boldsymbol{F}_{\mu\nu} &= \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + g\boldsymbol{A}_\mu \times \boldsymbol{A}_\nu \\ &= \frac{1}{g}(\partial_\mu \boldsymbol{\phi} \times \partial_\nu \boldsymbol{\phi} + f_{\mu\nu}\boldsymbol{\phi}) , \end{aligned} \tag{9}$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_{\mu'}$$

$$F_{\mu\nu} \equiv \boldsymbol{\phi} \cdot \boldsymbol{F}_{\mu\nu} = \frac{1}{g}(\boldsymbol{\phi} \cdot \partial_\mu \boldsymbol{\phi} \times \partial_\nu \boldsymbol{\phi} + f_{\mu\nu}) .$$

The gauge field equations

$$D_\mu \boldsymbol{F}_{\mu\nu} = \boldsymbol{j}_\nu , \tag{10}$$

where the source $\boldsymbol{j}_\nu$ comes from fields other than $\boldsymbol{\phi}$ and $\boldsymbol{A}_\mu$, are equivalent to a U(1) equation

$$\partial_\mu F_{\mu\nu} = j_\nu , \qquad j_\nu = \boldsymbol{\phi} \cdot \boldsymbol{j}_\nu \tag{11}$$

because $\boldsymbol{F}_{\mu\nu} // \boldsymbol{\phi}$, $D_\mu \boldsymbol{\phi} = 0$.

3. String-monopole solution in a SU(2) × U(1) model

We apply the method of sect. 2 to the Weinberg-Salam model. In this case ϕ is an isodoublet, for which we write

$$D_\mu \phi = (\partial_\mu - i\,\tfrac{1}{2} g\tau^i \cdot A_\mu{}^i - i\,\tfrac{1}{2} g' A_\mu{}^0)\phi = 0 . \tag{12}$$

Again ϕ may be normalized to unity:

$$(\phi^\dagger\phi) = \sum \phi^{\dagger\alpha}\phi^\alpha = 1 \ . \tag{13}$$

Furthermore, there are a series of Fierz identities

$$\begin{aligned}
&\sum_i (\phi^\dagger\tau^i\phi)^2 = (\phi^\dagger\phi)^2 \ , \\
&\sum_i (\phi^\dagger\tau^i\phi)(\phi^\dagger\tau^i\partial_\mu\phi) = (\phi^\dagger\phi)(\phi^\dagger\overleftrightarrow{\partial_\mu}\phi) \ , \\
&(\phi^\dagger\tau^i\phi)(\phi^\dagger\overleftrightarrow{\partial_\mu}\phi) - (\phi^\dagger\phi)(\phi^\dagger\tau^i\overleftrightarrow{\partial_\mu}\phi) = i(\phi^\dagger\tau^i\phi)\partial_\mu(\phi^\dagger\tau^k\phi)\epsilon^{ijk} \ , \text{ etc.}
\end{aligned} \tag{14}$$

These useful identities can be derived with the aid of the Dirac exchange operator.

It is not difficult to solve eq. (12) for the potentials. We find

$$gA_\mu{}^i + g'A_\mu{}^0(\phi^\dagger\tau^i\phi) = -i(\phi^\dagger\tau^i\overleftrightarrow{\partial_\mu}\phi) \ , \tag{15}$$

or, in view of eq. (14),

$$= -(\phi^\dagger\tau^j\phi)\partial_\mu(\phi^\dagger\tau^k\phi)\epsilon^{ijk} - i(\phi^\dagger\tau^i\phi)(\phi^\dagger\overleftrightarrow{\partial_\mu}\phi) \ , \tag{16}$$

which can be split into two equations

$$\begin{aligned}
&gA_{\mu\perp}^i = -(\phi^\dagger\tau^j\phi)\partial_\mu(\phi^\dagger\tau^k\phi)\epsilon^{ijk} \ , \\
&gA_{\mu\parallel}^i + g'A_\mu{}^0 = -i(\phi^\dagger\overleftrightarrow{\partial_\mu}\phi) \ ,
\end{aligned} \tag{17}$$

with $A_\perp$ and $A_\parallel$ referring respectively to the components perpendicular and parallel to $\phi^\dagger\tau^i\phi$. We will therefore parametrize the general solution as *

$$\begin{aligned}
&gA_{\mu\perp}^i = -(\phi^\dagger\tau^j\phi)\partial_\mu(\phi^\dagger\tau^k\phi)\epsilon^{ijk} \ , \\
&gA_{\mu\parallel}^i = -\xi i(\phi^\dagger\tau^i\phi)(\phi^\dagger\overleftrightarrow{\partial_\mu}\phi) \ , \\
&g'A_\mu{}^0 = -\eta i(\phi^\dagger\overleftrightarrow{\partial_\mu}\phi) \ , \qquad \xi + \eta = 1 \ .
\end{aligned} \tag{18}$$

Note the similarity between eqs. (8) and (18); $(\phi^\dagger\tau^i\phi)$ plays the role of the normalized isovector field, and obviously satisfies $D_\mu(\phi^\dagger\tau^i\phi) = 0$.

* Actually, we may add to $gA_{\mu\parallel}^i$ and $g'A_\mu{}^0$, terms proportional to an arbitrary potential $\mathcal{A}_\mu$ and still satisfy eq. (17). Here, we drop these "external" potentials.

From eq. (18) we compute the fields. After some algebra the result may be cast in alternative forms related by the Fierz identities:

$$g'F_{\mu\nu}{}^0 = \eta f_{\mu\nu} ,$$

$$gF_{\mu\nu}{}^i = -\partial_\mu(\phi^\dagger\tau^j\phi)\partial_\nu(\phi^\dagger\tau^k\phi)\epsilon^{ijk} + \xi f_{\mu\nu}(\phi^\dagger\tau^i\phi) ,$$

$$f_{\mu\nu} = -2i(\partial_\mu\phi^\dagger\partial_\nu\phi - \partial_\nu\phi^\dagger\partial_\mu\phi) ,$$

or

$$gF_{\mu\nu}{}^i = -(1-\xi)f_{\mu\nu}(\phi^\dagger\tau^i\phi) = -\eta f_{\mu\nu}(\phi^\dagger\tau^i\phi) , \tag{19}$$

so that we have always

$$gF_{\mu\nu}{}^i + g'F_{\mu\nu}{}^0(\phi^\dagger\tau^i\phi) = 0 . \tag{20}$$

This in turn means

$$|gF_{\mu\nu}| = |g'F_{\mu\nu}{}^0| ,$$

$$gF_{\mu\nu}{}^i(\phi^\dagger\tau^i\phi) + g'F_{\mu\nu}{}^0 \equiv \sqrt{g^2+g'^2}\,F_{\mu\nu}{}^Z = 0 . \tag{21}$$

Here $F_{\mu\nu}{}^Z$ stands for the field associated with the neutral massive boson Z^0, defined in a covariant way. The electromagnetic field $\mathcal{F}_{\mu\nu}$ should then be identified with

$$\mathcal{F}_{\mu\nu} \equiv [g'F_{\mu\nu}{}^i(\phi^\dagger\tau^i\phi) - gF_{\mu\nu}{}^0]/(g^2+g'^2)^{1/2}$$

$$= [(g^2+g'^2)^{1/2}/g]F_{\mu\nu}{}^0 . \tag{22}$$

Let us now assume for ϕ the form mentioned before,

$$\phi = \begin{pmatrix} \cos\frac{1}{2}\theta \\ \sin\frac{1}{2}\theta\, e^{i\varphi} \end{pmatrix} . \tag{23}$$

Because of the phase factor, ϕ has a line of singularity along $x = y = 0, z \leqslant 0$, but $(\phi^\dagger\tau^i\phi)$ is regular except at the origin. This means that $A^i_{\mu\perp}$ is always free of the line singularity. In fact we find

$$(\phi^\dagger\tau^i\phi) = x^i/r , \qquad gA_k{}^0 = -\epsilon^{kl3}x_l/r(r+z) ,$$

$$gA^i_{k\perp} = \epsilon^{ikl}x_l/r^2 , \qquad f_{kl} = \epsilon^{klm}x_m/r^3 . \tag{24}$$

The last line shows that $F_{\mu\nu}{}^i$ and $F_{\mu\nu}{}^0$ are precisely monopole fields, and therefore a solution of field equations except for the singularities. The line singularity is manifest only in ϕ, $A_\mu{}^0$ and $A_\mu{}^i_\parallel$. To grasp the physical significance of this fact, compute the magnetic flux out of the origin, by using the Gauss theorem for $F_{\mu\nu}{}^0$ and $F_{\mu\nu}{}^i(\phi^\dagger\tau^i\phi)$, and also compute the singular flux through the negative z axis by using the Stokes theorem for $A_\mu{}^0$ and $A_\mu{}^i$. The former computation yields

$$4\pi\eta/g' \quad \text{and} \quad -4\pi\eta/g \tag{25}$$

for U(1) and SU(2) parts respectively, and the latter computation yields

$$-4\pi\eta/g' \quad \text{and} \quad -4\pi\xi/g\,. \tag{26}$$

The former has missed the singular contributions, so the total flux out of the origin should be the sum of the two, namely

$$\text{zero for U(1) and } -4\pi/g \text{ for SU(2)}\,. \tag{27}$$

The emerging picture may then be stated as follows. The U(1) flux is sourceless, like that of a solenoid. The flux $4\pi\eta/g'$ that spreads out to infinity is returned surreptitiously through the string or tube along the negative z axis. The SU(2) flux, on the other hand, forms a genuine monopole of the correct quantized value $4\pi/g$, part of which spreads out and the rest goes through the string. There is one parameter $\xi = 1 - \eta$ which determines the division, and it so far remains arbitrary.

Another way to put it would be to say that the spreading field is an electromagnetic (since $F_{\mu\nu}{}^Z = 0$) field due to a magnetic charge

$$4\pi\eta(g^2 + g'^2)^{1/2}/gg' = 4\pi\eta/e \equiv Q\,, \tag{28}$$

whereas the flux tube contains a mixture of both,

$$4\pi\left(-\frac{g'}{g}\xi + \frac{g}{g'}\eta\right)\Big/\sqrt{g^2+g'^2} = (4\pi/e)[-\xi\sin^2\theta + \eta\cos^2\theta] \qquad \text{for } \mathcal{F}_{\mu\nu}\,,$$

$$-4\pi/\sqrt{g^2+g'^2} = -(4\pi/e)\sin\theta\cos\theta \qquad \text{for } F_{\mu\nu}{}^Z\,, \tag{29}$$

where θ now stands for the Weinberg angle.

To proceed further, we must resort to the dynamics of the original field theory. Then the singularities will be smeared out, and the total energy will be minimized. Since the flux of the Z field through the tube is fixed by quantization, this means that the energy of the tube per unit length will be minimum if the electromagnetic flux is zero, or in other words

$$\xi = \cos^2\theta\,, \qquad \eta = \sin^2\theta\,. \tag{30}$$

The monopole will then carry a magnetic charge

$$Q_0 = (4\pi/e)\sin^2\theta\,. \tag{31}$$

The system still has an infinite energy, and will not be stationary: the infinitely long string will keep pulling the monopole with a constant force. But a quasi-stationary finite energy system can be formed by spinning a pair of monopoles joined by a string. A mathematical description of such a dumb bell system [10,11] (monopolium) * is relatively simple only in the classical string picture. We will not discuss in detail how to carry out the transition to such a picture except to remark that the Higgs field ϕ for a pair of monopole and antimonopole placed at two fixed positions on the z axis is given by [8]

$$\phi(x) = \begin{pmatrix} \cos \frac{1}{2}\Theta \\ \sin \frac{1}{2}\Theta \, e^{i\varphi} \end{pmatrix} , \tag{32}$$

$$\cos \Theta = \cos \theta_1 - \cos \theta_2 + 1$$

instead of eq. (23). Here $\theta_1(\theta_2)$ is the polar angle of point x as seen from the monopole (antimonopole). The line singularity along the z axis now exists only between the poles. Far away from the system, $\cos \Theta \sim 1$ in all directions, i.e., ϕ approaches the standard constant solution, as it should.

4. Dynamics of a rotating dumb bell

We will skip any attempt to solve the field equations near the singularities because exact solutions cannot be obtained. Assuming that such solutions exist, however, it is easy to make estimates for the string tension and the monopole mass. The relevant part of the Weinberg-Salam Lagrangian is

$$-L = \tfrac{1}{4}F_{\mu\nu}{}^i F_{\mu\nu}{}^i + \tfrac{1}{4}F_{\mu\nu}{}^0 F_{\mu\nu}{}^0 + \tfrac{1}{2}(D_\mu \phi^\dagger)(D_\mu \phi) + \tfrac{1}{8}\lambda^2(\phi^\dagger\phi - 1)^2 , \tag{33}$$

where the energy is measured in units of $\langle\phi\rangle = 246$ GeV. This unit of energy will be adopted from here on. The W-boson and Higgs boson masses are respectively

$$m_{\mathrm{W}} = \tfrac{1}{2}g , \qquad m_{\mathrm{H}} = \lambda . \tag{34}$$

Consider first the flux tube between two poles. We approximate it by a cylinder of radius ρ and length $l >> \rho$, within which $\phi = 0$ and the fluxes given by eqs. (25), (26) are uniformly distributed. Minimizing the total energy with respect to ρ, we find the tension τ of the tube (string)

$$\tau = 2\lambda\pi^{1/2} [\xi^2/g^2 + \eta^2/g'^2]^{1/2} , \tag{35}$$

* This name is due to Maki [12], who has considered similar configurations in superfluid ^{3}He.

with

$$\rho = (8/\lambda)^{1/2}(\xi^2/g^2 + \eta^2/g'^2)^{1/4} .$$

Minimizing this further with respect to ξ, we recover the condition (30) and

$$\tau_0 = (2\lambda\pi/e) \sin\theta \cos\theta = \pi \cos\theta (m_{\mathrm{H}}/m_{\mathrm{W}}) ,$$

$$\rho_0 = ((8/\lambda e) \sin\theta \cos\theta)^{1/2} = 2((m_{\mathrm{W}} m_{\mathrm{H}})^{-1} \cos\theta)^{1/2} . \tag{36}$$

To estimate the monopole mass M, we assume a hollow sphere of radius r inside which all the fields are zero. Minimizing with respect to r gives

$$M = (8\pi/3e)(\lambda/2e)\eta^{3/2} , \qquad r = (2\eta/\lambda e)^{1/2} , \tag{37}$$

and for

$$\eta = \sin^2\theta , \quad M_0 = (4\pi/3e)(\sin\theta)^{5/2}(m_{\mathrm{H}}/m_{\mathrm{W}})^{1/2} ,$$

$$r_0 = ((m_{\mathrm{H}} m_{\mathrm{W}})^{-1} \sin\theta)^{1/2} . \tag{38}$$

The energy of a pair of monopoles separated by a distance l is then $2M - Q^2/4\pi l$, but the second term may be ignored if the poles are well separated from each other.

With τ_0 and M_0 fixed by eqs. (36) and (38), the total energy and angular momentum of a rotating dumb bell are computed to be

$$E \sim \tfrac{1}{2}\pi l \tau_0 , \qquad L \sim \tfrac{1}{8}\pi l^2 \tau_0 , \qquad l\tau_0/2M_0 = \beta^2/(1-\beta^2) , \tag{39}$$

where β is the velocity of the poles. This amounts to a straight Regge trajectory

$$L \sim \alpha_0' E^2 , \qquad \alpha_0' = 1/2\pi\tau_0 . \tag{40}$$

We have shown here only asymptotic relations which are valid in the relativistic limit $\beta \simeq 1$. In non-asymptotic cases, not only is the trajectory non-linear, but also the magnetic charge (visible from the outside) becomes L dependent, because the parameter ξ has to be chosen to minimize the total energy.

Let us now make rough numerical estimates. From eqs. (36) and (38) we obtain

$$(1/\alpha_0')^{1/2} = (2\pi\tau_0)^{1/2} \simeq \sqrt{2}\pi((m_{\mathrm{H}}/m_{\mathrm{W}}) \cos\theta)^{1/2}$$

$$= 1.1 \text{ TeV} \times ((m_{\mathrm{H}}/m_{\mathrm{W}}) \cos\theta)^{1/2} ,$$

$$M_0 \simeq (4\pi/3e)(\sin\theta)^{5/2}(m_{\mathrm{H}}/m_{\mathrm{W}})^{1/2}$$

$$= 3.4 \text{ TeV} \times (\sin\theta)^{5/2} (m_H/m_W)^{1/2} . \tag{41}$$

Unless m_H/m_W is unusually small, we are dealing with multi-TeV phenomena. For example, with $\sin^2\theta = \frac{1}{3}$, $m_H/m_W = 1$, eq. (41) gives $(1/\alpha_0')^{1/2} = 1$ TeV, $M = 0.9$ TeV.

On the other hand, the dimensions of the string and monopoles are of the order of $(1/m_W m_H)^{1/2}$, which would be in the $(100 \text{ GeV})^{-1}$ range. So our strings and poles are very fat in comparison with their mass scale. This is due to the smallness of the coupling constants. The object would not look like a rod unless $l >> \rho$, which translates into

$$L >> \tfrac{1}{2}\pi \times 137 \sin^2\theta \cos\theta . \tag{42}$$

The right-hand side of the above relation is 70 for $\sin^2\theta = \frac{1}{3}$. Thus by the time the object takes on a well-defined linear shape, it really behaves like a classical relativistic system.

How stable and well defined will such an object be once it is formed? Because the coupling constants are small, we expect that the decay widths as well as the quantum corrections to the classical solutions will be small. The system can decay by emitting γ's, W's and Z's, pairs of leptons and quarks, or by breaking up. The last process becomes more difficult for higher L because of energetics and angular momentum barrier, but it also means that production of such a state would be more difficult.

As an example, we estimate the radiative energy loss of rotating magnetic poles in classical electrodynamics. The process is the familiar synchrotron radiation. The radiated power is given by

$$P \sim \tfrac{8}{3}(Q^2/4\pi)\gamma^4/l^2 = \tfrac{8}{3} \times 137 (\tau_0/M_0)^2 \sin^4\theta$$

$$\simeq (m_H/m_W)\,\text{TeV}^2 \qquad \text{for} \qquad \sin^2\theta = \tfrac{1}{3} . \tag{43}$$

P is related to the radiative width Γ by $\Gamma = P/E$, and the measure of stability Γ/E takes the simple form

$$\Gamma/E = P/E^2 \simeq 1/L . \tag{44}$$

For the range of L we are considering, the system is indeed rather stable.

5. Discussion

The arguments and calculations presented here are certainly not rigorous, but the existence of massive string-like solutions in the framework of Weinberg-Salam theory seems very plausible. As already remarked, the smallness of the coupling constant makes the mass scale uncomfortably large, but at the same time it keeps the quantum effects small, once a well defined string-like state is formed. For this to be the case, the angular momentum has to be large, $L \gtrsim 100$, and the energy $E \gtrsim 10$ TeV. It is a simple consequence of the fact that the size of a monopole is ~137 times its own Compton wave length.

A pertinent question would be the production mechanism and the magnitude of cross section. In principle, a 10 TeV object may be produced in a proton-proton collision with projectile energy of 10^5 TeV and impact parameter of $10^{-4}\ m_W^{-1}$. At such an impact parameter, the electromagnetic interaction between point-like quarks would be $\simeq \frac{1}{137} \times 10^4\ m_W \simeq 10^2\ m_W$, which is of the order of the mass to be produced.

In addition to the rotating states, there must also exist longitudinal vibrations [10,11] of the monopoles. We did not discuss these states before because of the theoretical uncertainties when the poles overlap. They are probably very unstable and ill-defined. However, the production of a spin-1 rate through one-photon annihilation of an electron and a positron, for example, is forbidden because in the c.m.s. the $e^- - e^+$ pair converts into a purely electric field, whereas the monopole pair converts into a purely magnetic field.

As a final remark we emphasize the extreme similarity between the present results and the string model of mesons. If we scale down the masses by 1000, we obtain almost exact analogs of the mesons except for an absence of quarks and associated quantum numbers. It remains to be seen whether this is more than an analogy.

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