Chapter 28

INCENTIVE ASPECTS OF DECENTRALIZATION*

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1. Resource allocation as information processing

A resource-allocation mechanism is sometimes viewed as a gigantic information processing system. Such a system utilizes the knowledge dispersed among economic agents concerning their preferences, technologies, and endowments in order to determine how resources should flow. Information is transmitted between economic units and processed by them through computations which result in allocative decisions. Alternative mechanisms can then be compared in terms of their efficiency in processing information adequate for optimal decisions. It is from this point of view that Hayek (1945) stressed the merits of the competitive-market mechanism. The informational aspects of resource-allocation mechanisms were formalized and analyzed in a number of papers in the last twenty-five years [Hurwicz (1960), Mount and Reiter (1974), Reiter (1974a, 1974b), Hurwicz (1972, 1977), Walker (1977), Osana (1978), Sato (1981), and Jordan (1982)].

From an informational point of view an economic mechanism may be thought of as an exchange of messages. In line with the tâtonnement idea, an outcome is determined when the exchange of messages is in a stationary position.

Denote by $\mathscr{M}^i$ the "language" of the messages to be used by agent i. Then the process of exchanging messages may be represented by a system of difference equations of the form

$$m_{t+1}^i = f^i(m_t^1, \ldots, m_t^n; e), \qquad t = 0, 1, \ldots, \qquad i \in \{1, \ldots, n\} \equiv N,$$

where n is the number of agents, e represents the *economic environment* (preferences, endowments, technologies) and $m_t^i \in \mathscr{M}^i$ for all $t = 0, 1, \ldots,$ and all $i \in N = \{1, \ldots, n\}$.[1]

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[1] f^i is called agent i's *response* function.

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A message n-tuple $\bar{m} = (\bar{m}^1, \ldots, \bar{m}^n)$ is stationary if it satisfies the equation system

$$\bar{m}^i = f^i(\bar{m}^1, \ldots, \bar{m}^n; e), \qquad i \in N. \tag{1.1}$$

Denote by Z the space of conceivable resource allocations. Then, given the stationary message n-tuple $\bar{m}$, the resulting allocation $\bar{z}$ is determined by the outcome function $h: \mathscr{M} \to Z$, where[2] $\mathscr{M} = \mathscr{M}^1 \times \cdots \times \mathscr{M}^n$, so that

$$\bar{z} = h(\bar{m}).$$

The response functions f^i are assumed to be defined for environments e which are elements of a class E of a priori admissible environments. An *adjustment process* for E is then defined as $\pi = (\mathscr{M}^1, \ldots, \mathscr{M}^n; f^1, \ldots, f^n; h)$.

Note that the ith equation of the system (1.1) may be interpreted as defining a correspondence $\mu^i: E \Rightarrow \mathscr{M}$ such that

$$m \in \mu^i(e), \qquad m = (m^1, \ldots, m^n),$$

if

$$m^i \in f^i(m^1, \ldots, m^n, e).$$

In turn, we also have the correspondence $\mu: E \Rightarrow \mathscr{M}$, defined by

$$m \in \mu(e) \quad \text{if and only if} \quad m \in \bigcap_{i \in N} \mu^i(e).$$

In this formulation a message n-tuple $\bar{m}$ is stationary (or: equilibrium) for e if and only if

$$\bar{m} \in \mu(e).$$

A mechanism is then defined as $(\mathscr{M}, \mu, h)$ where $\mu = \bigcap_{i \in N} \mu^i$ and $\mathscr{M}$ may, but need not, be a Cartesian product of some $\mathscr{M}^i$'s.

Historically, the economists' interest has been focused on informationally decentralized mechanisms. An adjustment process is said to be *privacy-preserving* if each response function f^i depends only on the characteristic e^i of the ith agent. Typically, e^i is defined in terms of the endowments, preferences, and technologies of that agent; e.g. $e^i = (\omega^i, R^i, T^i)$ where ω^i is the ith agent's initial endowment, R^i his/her preference relation, and T^i his/her technology. We may

[2] $\mathscr{M}$ is called the *message space*.

then write

$$m^i_{t+1} = f^i(m^1_t, \dots, m^n_t; e^i), \qquad t = 0, 1, \dots, \quad i \in N.$$

It is understood that $(e^1, \dots, e^n) = e$.

A corresponding formulation of the privacy-preserving property in terms of the equilibrium correspondences is

$$\mu(e) = \bigcap_{i \in N} \mu^i(e^i),$$

where, this time, $\mu^i: E^i \Rightarrow \mathscr{M}$, and E^i is the a priori admissible domain of characteristics of the ith agent. Naturally, we have $E = E^1 \times \cdots \times E^n$.

A *message mechanism* $\chi = (\mathscr{M}, \mu, h; E, Z)$ on the class of environments E to an outcome space Z defines a correspondence, say $F: E \Rightarrow Z$, by

$$F(e) = \{z \in Z: z = h(m), m \in \mu(e) \text{ for some } m \in \mathscr{M}\},$$

for every e in E. F is said to be the (*static*) *performance correspondence* of the message mechanism.[3] Given a correspondence $G: E \Rightarrow Z$ and a message mechanism χ, we say that χ *realizes* G *over* E if $F(e) \neq \emptyset$ and $F(e) \subseteqq G(e)$ for all e in E. We say that χ *fully realizes* G *over* E if $F(e) \neq \emptyset$ and $F(e) = G(e)$ for all e in E.

It is important to note that the static aspects of performance depend on the response functions but only through the equilibrium message correspondence μ. I.e. if two n-tuples of response functions generate the same μ, then the resulting performance correspondence will be the same.

Much of the traditional welfare economics takes a mechanism (e.g. perfect competition) as given and investigates the properties (e.g. Pareto-optimality) of its (static) performance correspondence.

More recently, the reverse problem has come under investigation: given a correspondence $F: E \Rightarrow Z$, viewed as a social desideratum, are there mechanisms which (fully) realize it? In particular, one question has been whether there exist *decentralized* (privacy-preserving) mechanisms realizing the Pareto correspondence over "non-classical" sets of environments, such as those with indivisibilities, non-convexities, externalities, etc., where the competitive mechanism is known to fail. On the other hand, for the "classical" environments, in which the competitive mechanism is known to realize the Pareto correspondence, the problem has been whether there exist mechanisms that are informationally equally or more efficient (e.g. with message spaces of lower dimension) but that

[3] Mount and Reiter (1974) and Reiter (1977).

still realize the Pareto correspondence. The realization of other correspondences (e.g. those that are individually rational or envy free) has also been studied.

2. Decentralization in economies with public goods

The preceding analysis treats the economic agent as an information processing (communicating, computing) unit. But this, of course, is very inadequate, since economic agents also have motivations and preferences and their levels of satisfaction depend on the allocative decisions. Because of the dispersion of knowledge, it is usually possible for an economic agent to transmit false information. If this possibility is ignored in the design of a mechanism, the system is likely to malfunction. In early discussions, this danger became particularly obvious in two situations: the behavior of enterprise managers in Lange–Lerner socialist economies,[4] and in the Lindahl scheme of allocation of public goods.[5] For our purposes, the latter problem constitutes a particularly convenient point of departure.

Consider an economy with two goods (a public good Y and a private good X) and n agents, $n \geqq 1$.[6] The ith agent's initial endowments are denoted by $\omega^i_X, \omega^i_Y \in R_+$, and the preferences are represented by a utility function[7] $u^i(x^i, y)$ which is strictly increasing in each of its arguments. (x^i is the total amount of private good available to agent i; y is the total amount of public good available to all agents.) The private good can be used as an input to produce the public good. This technology is expressed by the input requirements function g: it takes $g(y)$ units of X to produce y units of Y.[8] For instance, we may think of X as leisure–labor, and y as measuring the width of a road to be constructed; then $g(y)$ is the number of units of labor required to construct the road of width y. (We ignore the costs of maintenance and crowding effects.) Denote by t^i the work contribution by the ith agent, so that the amount of good X available to this agent after the contribution is

$$x^i = \omega^i_x - t^i.$$

An allocation $(x^1, \ldots, x^n, y)$ is *feasible* if

$$g(y) \leqq \sum_{i=1}^{n} t^i, \qquad t^i = \omega^i_x - x^i, \qquad i = 1, \ldots, n.$$

[4] Weitzman (1974), Fan (1975), Bonin (1975) and Gindin (1970).

[5] Lindahl (1919), Samuelson (1954), Malinvaud (1971), Drèze and de la Vallée Poussin (1969), and Groves and Ledyard (1977).

[6] The public goods aspect is trivial for $n = 1$.

[7] The representability of preferences by utility functions is convenient but not always essential. The analysis can often be carried out in terms of preference relations. [See e.g. Hurwicz (1979).]

[8] The function g will be called the *input requirement function*. It is the inverse of the production function for the public good.

The well-known Samuelson condition states that an interior allocation $(x^1,\dots,x^n, y)$ is Pareto-optimal for a differentiable utility function with $u^i_X > 0$ for all i and a differentiable input requirements function g only if

$$\sum_{i=1}^{n} \frac{u^i_y(x^i, y)}{u^i_x(x^i, y)} = g'(y),$$

where u^i_y, u^i_x are the partial derivatives with respect to x^i and y (marginal utilities) respectively. (Since $u^i_x > 0$ for all i, Pareto-optimality also requires that $\sum t^i = y$.)

A particularly simple case is obtained when constant returns prevail, so that $g(y) \equiv ky$ for some $k > 0$. It is then possible to choose the units of commodity measurement so that $k = 1$, so that $g(y) \equiv y$. In what follows we shall often confine ourselves to this case.

A number of allocation systems and equilibrium concepts for economies with public goods have been considered in the literature. These include the "equilibrium with subscription" [Malinvaud (1972, pp. 213–214)][9] and Foley's "public competitive equilibrium" [Foley (1970, p. 67) and Malinvaud (1972, pp. 215–218)]. The former is in general not Pareto-optimal. The latter is Pareto-optimal under classical assumptions, but it has been characterized by Milleron (1972, pp. 432, 453) as "an interpretation of Pareto optimum" rather than "a true definition of equilibrium"; as pointed out by Malinvaud, it is a partly cooperative solution, somewhat analogous to the concept of the core.

The best known concept of a decentralized Pareto-optimal equilibrium was proposed by Lindahl in 1919.[10]

Lindahl's "positive solution"[11] can be interpreted[12] in terms analogous to those involving the Walrasian auctioneer as follows. The auctioneer calls out a proposed price vector $(p_1,\dots,p_n)$ where p_i is the price to be paid for each unit of the public good Y by agent i. The agent treats the price parametrically, and calculates his demand $(\tilde{x}^i, \tilde{y}^i)$ by maximizing $u^i(x^i, y)$ with respect to its arguments subject to the budget condition $x^i + yp_i \leqq \omega^i_x + \omega^i_y p_i$. Equilibrium is obtained when all agents' demands for the public good are equal, i.e. $\tilde{y}^1 = \cdots = \tilde{y}^n$. Under classical assumptions, such equilibrium is Pareto-optimal.[13] However, as stressed by Samuelson[14] there is the problem of incentives: "It is in the interest of each person to give *false* signals, to pretend to have less interest in a given

[9]See also Milleron (1972, pp. 451–453), Groves and Ledyard (1977, p. 789, ex. 2.1), and Roberts (1976).
[10]Independently proposed by Bowen (1943).
[11]Of the 'just taxation' problem.
[12]See Johansen (1963).
[13]Foley (1970); and Milleron (1972).
[14]Samuelson (1954).

collective consumption activity than he really has, etc." Furthermore, the objections he raised applied not only to the Lindahl solution but to *any* decentralized solution. (In fact, the relevant section of Samuelson's paper is entitled "Impossibility of Decentralized Spontaneous Solution".[15]) Was Samuelson's impossibility claim correct? And was he right in contrasting[16] the impossibility when public goods are present with the self-policing nature of markets for private goods?

To try to answer these questions we shall first formulate them in an analytically tractable manner, with sufficient generality to apply both to public and private goods, and to a variety of mechanisms.

3. Mechanisms (game forms) and implementation of social choice rules

Let E^i, $i \in \{1,\dots,n\}$, $n \geqq 1$, denote the class of a priori admissible characteristics[17] for agent i, and $E = E^1 \times \cdots \times E^n$ denote the class of a priori admissible environments (economies). It is assumed that the E^i's, and hence E, are known to the designer. The designer also knows a social choice rule F, i.e. a correspondence[18] from E to the set $\mathscr{A}$ of conceivable[19] outcomes (resource allocations). The designer's task is to find a mechanism whose outcomes would, in some sense, implement (or be acceptable for) F.

But what is meant by a mechanism in this context? In this exposition, we shall think of the (*game*) *mechanism or game form* as an ordered pair (S, h) where $S = S^1 \times \cdots \times S^n$, S^i is the strategy domain of the ith agent, and h an *outcome function*[20] $h: S \to \mathscr{A}$. (We shall see subsequently how this game mechanism can be related to the notion of a message mechanism [an adjustment process] as defined above in Section 1.) We shall assume that the agents participate in a non-cooperative game, with the S^i's as their strategy domains and with the ith "payoff function" $v^i: S \to R$ defined by $v^i = u^i \circ h$, where u^i is the ith utility function $u^i: \mathscr{A} \to R$.[21] [Note that the domain of the utility function is the set $\mathscr{A}$ of conceivable outcomes. For selfish preferences this can be reduced to the ith agent's component z^i of the outcome $z = (z^1,\dots, z^n)$, $z \in \mathscr{A}$.]

More generally, instead of a utility function, one may merely postulate for each agent a preference ordering on $\mathscr{A}$, to be denoted by $\textcircled{\succsim}_i$ or R^i.

[15] See Samuelson (1955).

[16] Op. cit., p. 389.

[17] Typically, the characteristic e^i of the ith agent is given by $e^i = (C^i, \omega^i, R^i, Y^i)$ where C^i is the consumption set, ω^i the initial endowment, R^i the preference relation, and Y^i the production possibility description.

[18] By hypothesis, $F(e) \neq \emptyset$ for all $e \in E$.

[19] $\mathscr{A}$ includes all outcomes known to be possible, but may be broader.

[20] Called by Gibbard (1973) a "game form".

[21] R is the set of real numbers.

We define the *game* $(S; h, e)$ generated by the mechanism (S, h) in the environment e as the game $\Gamma = (S; \Pi)$ in which S^i is the strategy domain of the ith agent, $S = S^1 \times \cdots \times S^n$, $h: S \to R$ is an outcome function, and the *payoff relation* $\Pi_{h,e}$ of the ith player is given by

$$s' \Pi_{h,e} s'' \Leftrightarrow h(s') R^i(e) h(s'') \quad \text{for} \quad s', s'' \in S.$$

Here $R^i(e)$ is the weak preference relation of the ith agent in the environment e. E.g. in a pure exchange economy, if $\bar{e} = (\bar{e}^1, \ldots, \bar{e}^n)$, $\bar{e}^i = (\bar{\omega}^i, \bar{R}^i)$, then $\bar{R}^i = R^i(\bar{e})$.[22]

Then a list (n-tuple) of strategies $s^* \in S$, $s^* = (s^{*1}, \ldots, s^{*n})$, is called a *Nash equilibrium for the game* $(S; h, e)$ if, for each $i \in N$,[23]

$$h(s^*) R^i(e) h(s^*/s^i, i) \quad \text{for all} \quad s^i \in S^i,$$

where

$$(s^*/t, i) = (s^{*1}, \ldots, t, \ldots, s^{*n}),$$

with t in the ith place.[24] $h(s^*)$ is then called a *Nash equilibrium outcome* for $(S; h, e)$ (for a *Nash equilibrium allocation* when $\mathscr{A}$ is a space of allocations).

The set of all Nash equilibrium strategy lists for the mechanism (S, h) in the economy e is denoted by $\nu_{S,h}(e)$; it is a subset of S.

The set of all Nash equilibrium outcomes for the mechanism (S, h) in the economy e is denoted by $\mathscr{N}_{S,h}(e)$; it is a subset of $\mathscr{A}$. In fact, $\mathscr{N}_{S,h}(e) = h(\nu_{S,h}(e))$.

We say that a mechanism (S, h) *implements* or is *acceptable for*[25] the social choice rule F in the class of economies E, if, for all $e \in E$,

$$\mathscr{N}_{S,h}(e) \neq \emptyset, \tag{3.1}$$

and

$$\mathscr{N}_{S,h}(e) \subseteqq F(e). \tag{3.2}$$

[Recall that, by hypothesis, $F(e) \neq \emptyset$ for all $e \in E$.]

[22] $\bar{e}^i$ is called the ith agent's characteristic. $\bar{\omega}^i$ is that agent's initial endowment, and $\bar{R}^i$ his/her preference preordering (a total, reflexive, transitive binary relation). $a' R^i a''$ means that agent i either prefers a' to a'' or is indifferent between them.

[23] Where $N = \{1, \ldots, n\}$.

[24] Equivalently, $s^* \Pi_{h,e} (s^*/s^i, i)$ for all $s^i \in S^i$ and all $i \in N$.

[25] At times, to avoid ambiguity we use the expression Nash-implementation. In Hurwicz and Schmeidler (1978) a mechanism is called acceptable if, in the sense of the above definition, it is acceptable for the Pareto correspondence.

We say that the mechanism (S, h) *fully implements* the social choice rule F for the class of economies E if, for all $e \in E$,

$$\mathcal{N}_{S,h}(e) \neq \emptyset, \tag{3.1'}$$

and

$$\mathcal{N}_{S,h}(e) = F(e). \tag{3.2'}$$

I.e. the inclusion in (3.2) of the definition of acceptability becomes an equality. [Since $F(e) \neq \emptyset$ by hypothesis, (3.1′) follows from (3.2′), but we have shown it explicitly for the sake of parallelism with the previous definition.]

We may note that when F is a (single-valued) function, the two definitions coincide.

Denote by $P^i(e)$ the strict preference agent i in e, and by $A(e) \subseteq \mathcal{A}$ the set of outcomes possible when the environment e prevails. As usual, an outcome $\hat{z} \in \mathcal{A}$ is *Pareto-optimal in* e if

$$\hat{z} \in A(e),$$

and there is no $z' \in A(e)$ such that $z' R^i(e) \hat{z}$ for all $i \in N$ and $z' P^j(e) \hat{z}$ for some $j \in N$.

In the earlier period of investigation, the question asked was relatively modest. Given the class of a priori admissible economies E, does there exist a mechanism (S, h) such that, for each $e \in E$, every Nash equilibrium outcome is Pareto-optimal in e? To avoid triviality, it is natural to add the requirement that the set of Nash equilibria, and hence Nash outcomes, be non-empty for all $e \in E$. As above, when these two conditions are satisfied, we say that the mechanism (S, h) is Pareto-*acceptable* for E.

Groves and Ledyard, in their path-breaking contribution (1977), constructed a Pareto-acceptable mechanism for a wide class of economies,[26] with three or more agents, containing public goods. Hurwicz and Schmeidler (1978) and, independently, Maskin (1977) undertook a systematic study of the existence of Pareto-acceptable mechanisms.

Paradoxically, it was discovered that, for the case of two agents, $n = 2$, if all strict preference orderings were a priori admissible in E, then only dictatorial mechanisms were Pareto-acceptable for E. [See Theorem 1 and Corollary 1 in Hurwicz and Schmeidler (1978, p. 1451) and Theorem 1 in Maskin (1977).] By contrast, many non-dictatorial Pareto-acceptable mechanisms could be found when there were three or more agents [see the kingmaker outcome function in Hurwicz and Schmeidler (1978, p. 1452) and Example 1 in Maskin (1977)].

[26] For the nature of this class, see Groves and Ledyard (1980).

Once it became clear that, for $n > 2$, Pareto-acceptable mechanisms do exist, ambitions expanded. In particular, it was noted that in the Groves–Ledyard mechanism, an agent's utility at equilibrium could be lower than at the initial endowment,[27] i.e. that the requirement of individual rationality was violated. This was in contrast to the properties of such traditional equilibria as Walras or Lindahl. Both of these give each agent the option of not trading (i.e. staying at the initial endowment), hence they do satisfy individual rationality.

Formally, when E is a class of pure exchange economies with initial endowment allocation $\omega(e)$, a social choice rule $F\colon E \Rightarrow \mathscr{A}$ is *individually rational in E* if, for all $e \in E$, and every $z \in F(e)$,

$$z R^i(e)\, \omega(e) \quad \text{for all} \quad i \in N.^{28} \tag{3.3}$$

The same definition can be used in a more general setting, with $\omega(e)$ interpreted as some type of reference allocation (e.g. status quo). We shall denote by $I(e)$ the set of all allocations z satisfying (3.3).

4. Revelation mechanisms and dominance equilibria

The new problem then is whether, for a given class of environments E, there exists a mechanism (S, h) such that, for any $e \in E$ and any $z \in \mathscr{N}_{S,h}(e)$,

$$z \in P(e) \cap I(e),$$

where $P(e)$ and $I(e)$ are, respectively, the sets of Pareto-optimal and individually rational allocations in e.

This issue was already studied in Hurwicz (1972) in the context of Edgeworth Box examples (pure exchange, two goods, two persons). However, the formulation there introduced an additional requirement, which in present-day terminology could be expressed as the requirement that (S, h) be a revelation mechanism. A mechanism (S, h) is said to be a *revelation mechanism for* $E = E^1 \times \cdots \times E^n$, if $S^i = E^i$ for each $i \in N$, where E^i is the class of a priori admissible characteristics for the ith agent.[29] Hence a revelation mechanism for E can be written (E, h). A

[27] See Groves and Ledyard (1980, p. 1487).

[28] It is not obvious how to define individual rationality in an economy with production. For a possible interpretation of this concept, see Hurwicz (1979b, p. 159).

[29] This usage of the term "revelation mechanism" corresponds to that of Green and Laffont (1979, p. 50, definition 4.3) except that we use characteristics where they use valuation functions. Dasgupta, Hammond and Maskin (1979, p. 188) use the term "direct mechanism" in the same sense as our "revelation mechanisms".

revelation mechanism (E, h) for E is said to be *compatible with* (or *natural for*) social choice rule F, if for every $e \in E$,

$$h(e) \in F(e),$$

where F is the social choice rule to be implemented. (Note, it is legitimate here to have e as an argument of h because in a revelation game $S = E$.)

In particular, if the social choice rule F is a (single-valued) function, then

$$h(e) = F(e) \quad \text{for each} \quad e \in E,$$

and the revelation mechanism compatible with F becomes (E, F), since here $S = E$ and $h = F$.

Let the ith agent's *true* characteristic be denoted by $\dot{e}^i$. In games of revelation to be considered it is understood that

$$\dot{e}^i \in E^i \quad \text{for all} \quad i \in N.$$

This means that the true characteristic is a priori admissible (otherwise the designer is misinformed!) and also, since $S^i = E^i$, $i \in N$, that each agent has the option of using the true characteristic as his strategy. On the other hand, unless E^i is a one-element set (which case we shall exclude), he also has the option of using as his strategy some element $\tilde{e}^i$ of E^i which is different from $\dot{e}^i$.

This raises the question whether the true profile $\dot{e} = (\dot{e}^1, \ldots, \dot{e}^n)$ is a Nash equilibrium for the revelation mechanism (E, h). I.e. the question is whether, for each $i \in N$,

$$h(\dot{e}^1, \ldots, \dot{e}^n) R^i(\dot{e}) h(\dot{e}^1, \ldots, \tilde{e}^i, \ldots, \dot{e}^n) \quad \text{for all} \quad \tilde{e}^i \in E^i.$$

[Note that $R^i(e)$ depends on e^i only. In fact, usually, $e^i = (C^i, \omega^i, R^i, Y^i)$.]

We say [see Hurwicz (1972)] that a direct mechanism (S, h) is *incentive-compatible on E* if truth is always a Nash equilibrium in E, i.e. if

$$\nu_{S,h}(e) \neq \emptyset \quad \text{and} \quad e \in \nu_{S,h}(e) \quad \text{for all} \quad e \in E.$$

[A more stringent requirement would be $\{e\} = \nu_{s,h}(e)$ for all $e \in E$, i.e. that truth be the only Nash equilibrium.]

Now denote by $L(e)$ the set of Lindahl allocations[30] in the economy e. Then L is a social choice rule (the *Lindahl Social Choice Rule*). We may note for later

[30] See, e.g. Johansen (1963) and Hurwicz (1979a, 1979b). In the two-good economy described in Section 2, $e = (C^i, \omega^i, R^i)_{i \in N}$ and $L(e) = \{(\bar{x}^i, \bar{y})_{i \in N}$: (1) $(\bar{x}^i, \bar{y}) \in C^i$ for all $i \in N$, and (2) for some $(p_1, \ldots, p_n)$, and all $i \in N$, if $(x^i, y) \in C^i$ and $x^i + yp_i \leqq \omega^i_x + \omega^i_y p_i$, then $(\bar{x}^i, \bar{y}) R^i(e)(x^i, y)\}$.

reference that

$$L(e) \subseteq I(e),$$

i.e. a L is always individually rational,[31] and, under classical assumptions,

$$L(e) \subseteq P(e),$$

i.e. a Lindahl allocation is Pareto-optimal. To simplify exposition, let us suppose that, for each $e \in E$, there is a unique Lindahl allocation, also to be denoted by $L(e)$. Thus, now L is a function. Then the *Lindahl mechanism*, to be distinguished from the Lindahl social choice rule L, is the (unique) revelation mechanism natural for L, namely $(E; L)$.[32] I interpret Samuelson's critique as the claim that truth is not a Nash equilibrium for this game, i.e. that the Lindahl mechanism is not incentive compatible.

In a moment, we shall see that Samuelson's claim concerning the Lindahl mechanism was correct. But, somewhat surprisingly, it turned out that this was due not, as many thought, to the peculiar features of the public goods but rather to incentive problems that arise for private goods as well. This latter fact was shown in Hurwicz (1972). Consider a pure exchange economy with two *private* goods and two traders. First let the (revelation) mechanism require the agents to report their characteristics e^1, e^2; given these the outcome function dictates the Walrasian (competitive) allocation corresponding to $e = (e^1, e^2)$, to be denoted by $W(e)$.[33] Assume $\omega(e) \notin P(e)$, i.e. a non-optimal initial endowment. [For the sake of simplicity suppose that the competitive allocations are unique (i.e. that $W(e)$ is a singleton for all $e \in E$).] Then, if the a priori admissible class e is sufficiently rich, truth turns out not to be a Nash equilibrium. Thus the Walrasian (competitive) process, viewed as a revelation mechanism, was seen (in this special case) to be not incentive-compatible. But this fact turned out to be a special case of a more general phenomenon: for $n = 2$, pure exchange, two goods, any revelation mechanism guaranteeing Pareto-optimality and individual rationality[34] is not incentive-compatible if E is sufficiently rich.[35]

[31] Because it leaves each agent the freedom not to trade.

[32] This mechanism can be interpreted as follows. Each agent is asked to report his characteristic e^i. Given the reports $e^1, \dots, e^n$, the allocation prescribed by the mechanism is the Lindahl allocation for the economy in which $e^1, \dots, e^n$ are the agents' characteristics.

[33] I.e. we require the mechanism to be compatible with the Walrasian (competitive) social choice rule. In a pure exchange ("Edgeworth Box") economy where $e^i = (C^i, \omega^i, R^i)$, we have $W(e) = \{(\bar{z}', \dots, \bar{z}^n) : (1)\ \bar{z}^i \in C^i$ for each i, and (2) there exists $(p_1, \dots, p_n)$ such that, for all $i \in N$, if $z^i \in C^i$ and $p_1 z^i \leqq p_1 \omega^i$, then $\bar{z}^i R^i(e) z^i\}$.

[34] I.e. compatible with $P(e) \cap I(e)$.

[35] Although there are truthful equilibria when the initial endowment $\omega(e)$ is Pareto-optimal.

More precisely, for $n = 2$, pure exchange, two goods, if (S, h) is a revelation mechanism for E (i.e. $S = E$) and

$$\mathcal{N}_{S,h}(e) \subseteq P(e) \cap I(e) \quad \text{for all} \quad e \in E,$$

and if the class E is sufficiently rich, then

$$e \notin \nu_{S,h}(e),$$

unless $\omega(e) \in P(e)$.

Now, when there are two agents, the geometry of the Edgeworth Box so helpful for private goods has a close counterpart in the Kolm (equilateral) Triangle, using barycentric coordinates [Kolm (1964) and Malinvaud (1971)]. Using this fact, Ledyard and Roberts (1974) showed that the phenomenon just described for a private goods economy also occurs in a two-agent economy with two goods, one of which is public, and where constant returns prevail in producing the public good, using the private good as input.[36]

Thus they showed that for such public goods economies ($n = 2$), all revelation mechanisms guaranteeing Pareto-optimality and individual rationality over a sufficiently rich class of environments, are not incentive-compatible. In particular, since the Lindahl mechanism does have these two properties, it is not incentive-compatible.[37]

Thus the Samuelson claim turns out to be correct, although for reasons not related to the presence of public goods![38]

The results so far discussed are very specialized. A number of generalizations have been obtained. The impossibility results just cited for $n = 2$ were extended by Ledyard (1977) as applied to "core-seeking" mechanisms for arbitrary n. See also Satterthwaite (1976) and Dasgupta, Hammond and Maskin (1979, p. 198 thru Section 4.4.1).[39]

[36] See Section 2. An analytic example is given in Roberts (1979, p. 289).

[37] In fact, the geometry of Lindahl equilibria in the Kolm Triangle is completely analogous to the geometry of the Walrasian (competitive) equilibria in the Edgeworth Box (price line, double tangency, etc.).

[38] However, the similarity between the public and private goods economies appears to break down as the number of agents $n \to \infty$. For private goods, Postlewaite and Roberts (1976) showed the competitive mechanism (which has individually rational equilibria) to be, in a sense, asymptotically incentive compatible. By contrast, Roberts (1976) found that, for public goods, mechanisms acceptable for individually rational choice rules (such as Lindahl) are *not* asymptotically incentive compatible. [However, when the individual rationality requirement is abandoned, asymptotic incentive compatibility is possible. This is shown to be the case in certain economies for the pivotal mechanism with rebates in Green, Kohlberg and Laffont (1976).]

[39] This theorem drops the requirement of individual rationality but requires a broader class E than used in Hurwicz (1972) and an additional condition for $n \geq 2$. See also Hammond (1979) for related results in "large" economies, i.e. those with a non-atomic measure of space agents.

Furthermore, it was discovered that there is a close relationship between incentive-compatibility as defined above and dominance-equilibria.

Let $\Gamma = (S, \pi)$ be a game with the strategy space $S = S^1 \times \cdots \times S^n$ and a payoff function $\pi = (\pi^1, \dots, \pi^n)$. Then s_i^* is said to be a *dominant strategy* for player i if

$$\pi^i\left(s_i^*, s_{)i(}\right) \geqq \pi^i\left(s_i, s_{)i(}\right) \quad \text{for all} \quad s_i \in S^i, \quad s_{)i(} \in S^{)i(},$$

where $S^{)i(} = S^1 \times \cdots \times S^{i-1} \times S^{i+1} \times \cdots \times S^n$. Similarly, when h is an outcome function, s_i^* is a dominant strategy for the game $(S; h, e)$ if

$$h\left(s_i^*, s_{)i(}\right) \; R^i(e) \; h\left(s_i, s_{)i(}\right) \quad \text{for all} \quad s_i \in S^i, \quad s_{)i(} \in S^{)i(}.$$

$s^* = (s_1^*, \dots, s_n^*)$ is said to be a *dominance equilibrium* if s_i^* is a dominant strategy for each $i \in N$.

The discovery was the following: if, for a direct revelation mechanism (E, h), truth is a Nash equilibrium for all $e \in E$, then truth is a dominant strategy for every agent.[40] Thus such truthful Nash equilibria are dominance equilibria, and a search for incentive-compatibility is a search for dominance equilibria.

Moreover, as shown by Dasgupta, Hammond and Maskin (1979, theorem 4.1.1, p. 194),[41] given a mechanism with dominance equilibria, there exists an equivalent[42] *direct* mechanism in which truthtelling constitutes a dominance equilibrium.

One approach, pioneered[43] by Vickrey (1961), Clarke (1971), and Groves (1979), is focused on the design of mechanisms for which truthtelling is a dominant strategy. It turns out that such mechanisms can be designed, at least for the class of "parallel" preferences, i.e. preferences representable by utility[44]

[40] See d'Aspremont and Gérard-Varet (1979a, theorem 1, p. 31) and Dasgupta, Hammond and Maskin (1979, theorem 7.1.1, pp. 209–210). In Gibbard (1973, p. 595) a game form is called *straightforward* if every player always has a dominant strategy (so a dominance equilibrium exists).

[41] The construction is used in Gibbard (1973, p. 596) and in Green and Laffont (1977, p. 434, proof of theorem 5).

[42] See Dasgupta, Hammond and Maskin (1979, p. 189). Let (S, g) be any mechanism possessing dominance equilibria in games $(S; h, e)$ for e in E, and let $s^* : E \to S$ be a (single-valued) selection from the dominance equilibrium correspondence. Then the composition $h = g \circ s^*$ defines the outcome function of the corresponding revelation mechanism. This mechanism (E, h) is said to be equivalent to (S, g) if, for all $e \in E$, $e \in \mathcal{N}_h(e)$, i.e. truth is an equilibrium for all environments. However, it may happen that (S, h) is compatible with F while (E, h) is not. [See Dasgupta, Hammond and Maskin (1979, p. 195).]

[43] With Jacob Marschak as a precursor; see Groves (1979, p. 50, footnote 11) and Green and Laffont (1979, p. 36).

[44] Called quasi-linear or transferable.

functions linear in the private good (which can be thought of as numéraire). Let the ith agent's preferences be represented by the transferable utility function $u^i(x^i, y) = x^i + v_i(y)$, where x^i is the amount of private good retained (after taxes) by i, and y the level of the public good available to everyone. $v^i(\cdot)$ is called the ith *valuation function*. [Our terminology differs somewhat from that used by Green and Laffont (1979) because they subsume the costs of a project in the valuation function.] Equivalently,

$$u^i = \omega_x^i - t^i + v_i(y),$$

where t^i is the tax (contribution) paid by the ith agent. For any individual agent this tax may be positive, negative, or zero. But if no outside funds are available [assuming the input requirements function $g(y) \equiv y$, i.e. constant returns prevail and measurement units are normalized (see Section 1)], taxes must satisfy the feasibility requirement

$$\sum_{i=1}^{n} t^i \geqq y.$$

Furthermore, an allocation is not Pareto-optimal unless the equality holds, i.e.

$$\sum_{i=1}^{n} t^i = y.$$

[See Groves (1979, p. 47, proposition 2, condition (a)). The notion of Pareto-optimality used by Groves and in our text differs from that in Green and Laffont (1979, p. 33) where the utility of the public agency [an additional $(n+1)$st agent] permits Pareto-optimality even when $\sum t^i > y$.]

If the valuation functions are concave, at a Pareto-optimal allocation, the expression

$$-y + \sum_i v_i(y)$$

must be maximized. (For differentiable v_i's and an interior optimum this yields the Samuelson condition in the form

$$\sum_i v_i'(y) = 1.)$$

On the other hand, if $\sum t^i = y$ and the expression $-y + \sum_i v_i(y)$ is being maximized, then the allocation is Pareto optimal.[45] Thus, for transferable utilities, the maximization condition determines the optimal values of the public good, independently of the distribution of tax burdens. If $\hat{y}$ is an optimal value of the public good, then an n-tuple $(t^1, \ldots, t^n)$ of taxes is Pareto-optimal if and only if it satisfies the conditions

$$\sum_{i=1}^{n} t^i = \hat{y},$$

and[46]

$$t^i \geqq -\omega_x^i \quad \text{for all} \quad i \in N.$$

Consider now a revelation mechanism with the initial endowments known to the designer. Under the assumptions made, the only aspect of the ith characteristic unknown to the designer is the ith valuation function, v_i. Therefore, the space E^i may be identified with a set V^i of the a priori admissible valuation functions for agent i.

An outcome function for such a mechanism is of the form

$$h: V^1 \times \cdots \times V^n \to R^n \times R_+,$$

so that

$$h: (v_1, \ldots, v_n) \mapsto (x^1, \ldots, x^n, y).$$

We shall write $x^i = h_{x_i}(v_1, \ldots, v_n)$, $y = h_y(v_1, \ldots, v_n)$.

Revelation mechanisms independently proposed by Clarke (1971) and Groves (1970) have the remarkable property that truthtelling is a dominance equilibrium when preferences are representable by utility functions linear in the private good.[47] Furthermore, these mechanisms have the following additional property.

Let w_i denote the valuation function *reported* (whether truthfully or not) by agent i. Then, in the above mechanism,

$$h_y(w_1, \ldots, w_n)$$

maximizes the expression

$$-y + \sum_i w_i(y)$$

[45]Assuming that, for each $i \in N$, $C^i = R_+^2$ and $t^i \geq -\omega_x^i$ ($C^i =$ the ith agent's consumption set). Cf. the corresponding theorem in Groves (1979, p. 47, proposition 2).

[46]Again assuming that each agent's consumption set $C^i = R_+^2$ (the non-negative quadrant).

[47]See Groves (1979, corollary, p. 51).

with respect to y. I.e. for all $(w_1,\ldots,w_n) \in V^1 \times \cdots \times V^n$,

$$h_y(w_1,\ldots,w_n) = \max_y \left[-y + \sum_i w_i(y) \right].$$

But truth constitutes a dominant strategy for each agent. Hence we have, at dominance equilibrium,

$$w_i = \mathring{v}_i \quad \text{for all} \quad i \in N,$$

where $\mathring{v}_i$ is the true valuation function. So, there exists a dominance equilibrium such that the value of $\hat{y}$ maximizes the expression $[-y + \sum_i \mathring{v}_i(y)]$, hence $\hat{y}$ is Pareto-optimal (with regard to the true preferences).

However, the fact that $\hat{y}$ is Pareto-optimal does not imply that the allocation $(x^1,\ldots,x^n,\hat{y})$ produced by these mechanisms is Pareto-optimal. As seen above, for Pareto-optimality it is necessary that

$$\hat{y} = \sum t^i = \sum \left(\omega_X^i - x^i \right). \tag{4.1}$$

The latter condition is not satisfied by the "*pivotal*" (Clarke) mechanism. Now the pivotal mechanism is a special case of a class called the *Groves mechanisms*. It turns out that, under certain conditions on the families V^i of a priori admissible valuations, the search for mechanisms whose dominance equilibria yield Pareto-optimal outcomes can be confined to Groves mechanisms.[48] But, unfortunately, it has been found that every Groves mechanism will violate the balance condition on large classes of profiles.[49]

[48]See Green and Laffont (1977, corollary 3, p. 433; 1979, theorem 4.5, pp. 63–64), Walker (1978), and Holmström (1979). A sufficient condition [Holmström (1979, theorem 2, p. 1141)] is that $V \equiv V^1 \times \cdots \times V^n$ be a convex set in the space of valuation n-tuples. This condition is satisfied in cases underlying the results due to Green and Laffont and to Walker where V_i consists of all continuous functions or all strictly concave (or convex) functions, or all concave quadratic functions on a convex subset of R^l.

Holmström (ibid., theorem 1, p. 1140) also shows a weaker condition (that V be "smoothly connected") to be sufficient.

On the relationship of the previous conditions, see Green and Laffont (1979, p. 65, footnote 15) and Holmström (1979, p. 1142).

[49]See Green and Laffont (1979, theorem 5.3, p. 90). Walker (1980, theorem 1, p. 1531) shows that the "failure set" is everywhere dense on a space of concave valuation functions when this space is endowed with any topology weaker than the strongest topology in which vector addition and scalar multiplications are continuous. Analogous results for private goods, pure exchange economies are presented in Hurwicz and Walker (1983). See also Hurwicz (1975a, 1975b, 1981) for impossibility results when $n = 3$.

However, the balance condition may be satisfied on sufficiently small classes of profiles. See Groves and Loeb (1975) and Hurwicz (1975a, 1975b, 1981). These cases involve economies with three or more agents. For impossibility results for two agents, see Green and Laffont (1979, p. 94) and Hurwicz (1975a, 1975b, 1981).

To present these results more formally, it is convenient to switch to a model of public decision-making which subsumes the inputs (costs) of producing a public good under the more general rubric of a public decision (project).[50]

Formally, this reduces the problem to that of a "costless" project, but at the expense of increasing the dimensionality of the space in which the project is defined.

Thus, let c^i, $c^i \leqq \omega^i_X$, $i=1,\dots,n$, be such that $\sum_{i=1}^n c^i = y$, $y \geqq 0$. Then the "project" $z=(c^1,\dots,c^n;\, y)$ requiring the ith agent to contribute c^i units of the private good to the production of y units of the public good is feasible. Suppose that proposal p combines this project with a transfer scheme $(r^1,\dots,r^n)$ where, for each $i \in N$, r_i is the compensation (in terms of the private good) paid to agent i. Then the utility of the proposal to the ith individual is $u^i[p] = u^i(\omega^i_X - c^i + r^i, y)$. When u^i is linear in the private good, we may write this as

$$u^i[p] = \omega^i_X - c^i + r^i + \Psi_i(y).$$

Alternatively, we can write

$$u^i[p] = \omega^i_X + r^i + \varphi_i(z), \qquad i \in N,$$

where

$$z = (c^1,\dots,c^n;\, y) \quad \text{and} \quad \varphi_i(z) = -c^i + \Psi_i(y).$$

Since $x^i = \omega^i_X - c^i + r^i$, $i \in N$, the feasibility condition,

$$y + \sum x^i \leqq \sum \omega^i_X,$$

becomes

$$y + \sum_i (\omega^i_X - c^i + r^i) \leqq \sum \omega^i_X.$$

Hence

$$y - \sum c^i + \sum r^i \leqq 0 \quad \text{or} \quad \sum c^i - y \geqq \sum r^i.$$

But, by construction, $\sum c^i - y = 0$. Therefore, the feasibility condition becomes

$$\sum_{i=1}^n r^i \leqq 0.^{51} \tag{4.2}$$

[50] See Green and Laffont (1979, pp. 29–31; p. 42, footnote 9; p. 52, footnote 5; pp. 74–75).

[51] Here and subsequently we shall ignore the implications of individual feasibility requirements, viz. that $x^i \equiv \omega^i_X - c^i + r^i \geqq 0$, $i \in N$.

Thus, in this formalization, the *public decision* (*project*) is a point $z = (c^1, \dots, c^n; y)$ in the $(n+1)$-dimensional Euclidean space R^{n+1} satisfying the condition

$$\sum_{i=1}^{n} c^i = y, \tag{4.3}$$

and a *proposal* is a point $p = (c^1, \dots, c^n; y; r^1, \dots, r^n)$ in the $(2n+1)$-dimensional space R^{2n+1} satisfying the conditions (4.2) and (4.3).

Somewhat more generally, and *reverting to the customary notation*,[52] a *proposal* is defined as a point $q = (r^1, \dots, r^n; y)$ where y is an element of a set Y of feasible projects and $\sum_{i+1}^{n} r^i \leqq 0$. As before, for preferences representable by utility functions of the form

$$u^i[q] = r^i + v^i(y), \qquad i \in N,$$

$v_i(\cdot)$ is again called agent i's *valuation function*.

The a priori admissible class of i's valuation function is denoted by V^i.

In a revelation game, an individual's strategy is an element w^i of V^i. A mechanism then is defined as a function

$$h: V^i \times \cdots \times V^n \to R^n \times Y,$$

so that

$$h(w_1, \dots, w_n) = (r^1, \dots, r^n, y) \quad \text{with} \quad \sum_{i=1}^{n} r^i \leqq 0.$$

Write $w = (w_1, \dots, w_n)$ and $r = (r^1, \dots, r^n)$. For $(r, y) = h(w)$, we shall use the notation $r^i = h^{ri}(w)$, $i \in N$, $r = h^r(y)$, and $y = h^y(w)$.

Example

Although we have come to this model through a re-formalization of a particular public goods problem, it has in fact much greater generality, as long as we retain the freedom of choosing a suitable set Y. In particular, a pure exchange economy with $k+1$ goods and selfish preferences can be modeled by choosing

$$Y = R^k \times \cdots \times R^k \quad (n \text{ times}).$$

Here, for $y \in Y$, we write $y = (y^1, \dots, y^n)$, $y^i \in R^k$, $i \in N$, and require that $\sum_{i=1}^{n} y^i \leqq 0$.

[52] Thus from now on y here corresponds to $z = (c^1, \dots, c^n; y)$ above (not to y above) and Y can be either multi- or one-dimensional; v_i here corresponds to Ψ_i above (not to v_i above).

Furthermore, the utility functions are made selfish by postulating that, for every $y \in Y$,

$$v_i(y) = \tilde{v}_i(y^i), \quad i \in N,$$

for some functions $\tilde{v}_1, \dots, \tilde{n}_n$.

Clearly, this represents a pure exchange model with (r^i, y^i) as the net trade of agent i and the agent's selfish utility function (in terms of net trades),[53]

$$u^i(r^i, y^i) = r^i + v_i(y^i).$$

Going back now to the general model, we note first that[54] a proposal $(\hat{r}^1, \dots, \hat{r}^n; \hat{y})$ is Pareto-optimal if and only if

$$\sum_{i=1}^{n} \hat{r}^i = 0, \tag{4.4}$$

and

$$\sum_{i=1}^{n} v_i(\hat{y}) \geqq \sum_{i=1}^{n} v^i(y) \quad \text{for all} \quad y \in Y. \tag{4.5}$$

Let $w_1, \dots, w_n$, $w_i \in V^i$, $i \in N$, be the agents' *reported* (possibly false) valuation functions.

A Groves mechanism uses an outcome function $h: V^1 \times \cdots \times V^n \to R^n \times Y$, such that (i)[55]

$$\sum_{i=1}^{n} w_i(h^y(w)) \geqq \sum_{i=1}^{n} w_i(y) \quad \text{for all} \quad y \in Y,$$

and (ii) for each $i \in N$, there exists a function $g_i: V^1 \times \cdots \times V^{i-1} \times V^{i+1} \to R$ such that

$$h^{ri}(w) = \sum_{j \neq i} w_i(h^y(w)) - g_i(w_{)i(}).$$

[53]See Walker (1980, section 5, p. 1534) and Walker (1977). I have also benefited from private communications with Walker on this subject.

[54]See Groves (1979, p. 47, proposition 2). Note again that the "if" part of the statement ignores the lower bound restrictions (due to considerations of individual feasibility) on the x^i.

[55]Writing $w = (w_1, \dots, w_n)$ and $w_{)i(} = (w_1, \dots, w_{i-1}, w_{i+1}, \dots, w_n)$.

I.e. the public decision $y^* = h^y(w)$ chosen by a Groves mechanism maximizes (over the set Y) the sum of *reported* valuations.[56] Clearly, if the reported valuations are truthful ($w_i = \dot{v}_i$ for all $i \in N$), then y^* will be a Pareto-optimal value of the public decision. Now,[57] truth is a dominant strategy for every agent. It follows that, for a Groves mechanism, there exist dominance equilibria yielding Pareto-optimal public decisions. I.e. if h is a Groves mechanism outcome function, then there exist some reported valuation lists $w^* \in V^1 \times \cdots \times V^n$, $\tilde{r} \in R^n$, $\sum_{i=1}^n \tilde{r}^i = 0$, such that (1) $w^* = \dot{v}$ (true valuation list), (2) w^* is a dominance equilibrium, and (3) $(\tilde{r}, h^y(w^*))$ is Pareto-optimal. However, this does not imply the optimality of $(r^*, y^*) = (h^r(w^*), h^y(w^*))$ because optimality requires that $\sum r^{*i} \leqq 0$, about which so far nothing has been said. (Note that $\tilde{r}$ above need not equal r^*.)

Example

As mentioned above, the pivotal (Clarke) mechanism is a Groves mechanism. To define it, it is sufficient to specify the "alien" component function g_i appearing in the definition of the compensation function h^{ri} for a Groves mechanism.

For each $i \in N$, let y_i^{**} denote the maximizer of $\sum_{j \neq i} w_j(y)$ over Y, i.e.

$$\sum_{j \neq i} w_j(y_i^{**}) \geqq \sum_{j \neq i} w_j(y) \quad \text{for all} \quad y \in Y.$$

Then, for the pivotal (Clarke) mechanism, we have

$$g_i(w_{)i(}) = -\sum_{j \neq i} w_j(y_i^{**}).$$

Thus, for this mechanism, the ith compensation function is given by

$$h^{ri}(w) = \sum_{j \neq i} w_j(y^*) - \sum_{j \neq i} w_j(y_i^{**}),$$

where y^* maximizes $\sum_{k=1}^n w_k(y)$ while y_i^{**} maximizes $\sum_{j \neq i} w_j(y)$; in both cases the maximization is over Y.

It is instructive to apply this in the special case where Y is a two-element set, say $Y = \{0,1\}$.[59] Normalize $w_i(\cdot)$ so that $w_i(0) = 0$, and write $w_i(1) = w_i$. Then[60]

[56] Revelation mechanisms with this property are called *direct* revelation mechanisms in Green and Laffont (1979, p. 51, definition 4.4).

[57] See Groves and Loeb (1975), Green and Laffont (1979, pp. 56–57, theorem 4.1).

[58] See Green and Laffont (1979, pp. 42–43; p. 52, definition 4.7).

[59] The interpretation is that 1 represents undertaking a specific project and 0 not undertaking it.

[60] See Green and Laffont (1979, p. 42).

the function h_i for the pivotal mechanism becomes

$$g_i(w_{)i(}) = \max\left(\sum_{j \neq i} w_j, 0 \right).$$

It turns out that the compensation r^i paid to agent i is either zero or $-\left|\sum_{j \neq i} w_j\right|$, and the latter can be the case for some agents[61] at a dominance (truthful) equilibrium. Hence it can happen at equilibrium that

$$\sum_{i=1}^{n} r^i < 0,$$

and this inequality implies absence of Pareto-optimality. Thus the pivotal mechanism does not guarantee the Pareto-optimality of the equilibrium allocation (r^*, y^*) it generates, although the choice of y^* itself is Pareto-optimal.[62]

Contrary to what one might have hoped, this difficulty is not remedied by broadening the horizon to the class of all Groves mechanisms. The fact that there exists no Groves mechanism such that the compensatory payments balance when the sets V^i are unrestricted, is demonstrated[63] in Green and Laffont (1979, p. 90, theorem 5.3). On the other hand, a balanced Groves mechanism[64] was constructed by Groves and Loeb (1975) for the case where $Y = \boldsymbol{R}_+$ and the valuation functions are quadratics,

$$v_i(y) = -\tfrac{1}{2}y^2 + \theta_i y, \qquad i \in N,$$

with $\theta_i \in \boldsymbol{R}_+$, provided $\#N \geqq 3$.

The requirement that $\#N \geqq 3$ is essential: when there are only two agents, balance cannot be achieved even for quadratic utility functions.[65]

Such results raise the question of conditions under which Groves mechanisms are balanced and hence Pareto-optimal. A necessary and sufficient differential

[61] It is the case for the "pivotal" agents. An agent i is "pivotal" if the sign of $\sum_{k=1}^{n} w_k$ is different from that of $\sum_{j \neq i} w_j$, namely where one of these sums is non-negative while the other is negative.

[62] In the sense that there exists some allocation $(\tilde{r}, y^*)$ which is Pareto-optimal. However, in general, $\tilde{r} \neq r^*$.

[63] Explicitly, for a two-element set Y.

[64] I.e. such that $h^r(w) = 0$ for all $w \in V^1 \times \cdots \times V^n$.

[65] Green and Laffont (1979, pp. 94–95).

condition for a class of differentiable valuation functions is given in Green and Laffont (1979, p. 96). It makes possible the direct testing of specific classes of environments.

So far we have been focusing on Groves mechanisms. But negative results concerning Groves mechanisms have broader implications. First, it has been shown[66] – under varying assumptions concerning the a priori admissible classes of valuations V^i, $i \in N$ – that Groves mechanisms are the only revelation mechanisms for which truth is always a dominant strategy. But even going beyond revelation mechanisms does not alter the situation materially as long as we insist on dominance equilibria. This is so because[67] given a mechanism (S, h), using an arbitrary strategy space $S = S^1 \times \cdots \times S^n$ and possessing a dominance equilibrium $s^* = (s_1^*, \ldots, s_n^*)$ over the environment $E = E^1 \times \cdots \times E^n$, there exists a corresponding revelation mechanism (E, g) in which truthtelling is dominant. Furthermore, the outcome generated by the truthtelling equilibrium corresponding to s^* yields the same outcome, hence preserves Pareto-optimality.[68]

Thus the difficulty arises as soon as one demands a mechanism with dominance equilibria. Such a mechanism will be subject to the same difficulties that would arise for Groves mechanisms. It has already been seen that there do exist[69] families of valuation functions for which balanced and truthtelling Groves mechanisms exist, hence Pareto-optimality can be guaranteed. But are such cases typical or exceptional? Answers to this question are provided in Walker (1980).

To formulate the relevant results one must specify the topology to be used on the space $V = V^1 \times \cdots \times V^n$ of valuation function profiles.

Walker requires V^i to be a subset of $C(\mathscr{A})$, the set of all continuous real-valued functions on $\mathscr{A}$ (the space of public decisions). Let $\bar{\mathscr{J}}$ be the largest topology in which $C(\mathscr{A})$ is a topological vector space, and let $\mathscr{J}$ be the topology on V^i inherited by V^i as a subspace of $C(\mathscr{A})$. ($\bar{\mathscr{J}}$ is called the vector space topology on V^i.)

Walker's Theorem 1 assumes $n \geqq 2$ and $\mathscr{A}$ a convex open set in R^l and uses $V^1 = \cdots = V^n$, with V^1 the set of all strictly concave valuations on $\mathscr{A}$ which attain a maximum on $\mathscr{A}$. Then, for any Groves mechanism, the set of profiles in $V^1 \times \cdots \times V^n$ for which the mechanism yields non-Pareto-optimal outcomes is everywhere dense[70] in $V^1 \times \cdots \times V^n$. The same proposition is valid for any mechanism where truth is a dominant strategy.

[66] Green and Laffont (1977, theorem 3, corollary 3; 1979, theorem 4.5, pp. 63–64), Walker (1978), and Holmström (1979). See footnote 49.

[67] See footnote 41.

[68] See, however, Dasgupta, Hammond and Maskin (1979, pp. 189, 194–195) concerning complications due to additional, possibly non-optimal, equilibria.

[69] Quadratic in the public good, linear in the private good (see above).

[70] In the product topology $\bar{\mathscr{J}}^{(n)}$ or in any topology $\mathscr{J}^{(n)}$ such that $\mathscr{J} \subset \bar{\mathscr{J}}$.

Now, as mentioned above, mechanisms with dominance strategies can be transformed into revelation mechanisms where truthtelling is dominant. Given a mechanism (S, h) for E with a dominance equilibrium $(s_1^*, \dots, s_n^*)$, we define a mechanism (E, g) with truthtelling dominance equilibria, where

$$g(e) = h\left(s_1^*(e^1), \dots, s_n^*(e^n)\right),$$

and e^i is the characteristic of the ith agent.

This transformation yields a strengthening of Walker's Theorem 4 which, under the same assumptions on $\mathscr{A}$ and the V^i's, asserts that any mechanism yielding dominance equilibria will have an everywhere dense set where optimality fails.

However, a stronger conclusion is obtained if the mechanism yielding dominance equilibria is *continuous* and each V^i a compact subset of $C(Y)$. In that case, Walker's Theorem 5 states that the ("good") set of profiles with Pareto-optimal outcomes is closed and nowhere dense in $V^1 \times \cdots \times V^n$. Thus here the failure set is open as well as everywhere dense.

The results so far discussed show that, generally speaking, one cannot hope for both dominance equilibria and Pareto-optimality. Several alternative directions of research have been explored, including asymptotic properties of mechanisms as the number of agents tends to infinity,[71] sampling procedures,[72] and Bayesian specifications.[73]

These directions attempt to preserve, to the extent possible, the availability of dominance or truthful equilibria, possibly at the expense of other desirable properties of mechanisms.

5. Pareto-optimal Nash equilibria in economies with public goods

On the other hand, one may sacrifice the dominance equilibria and accept the weaker type, namely the non-cooperative Nash equilibria discussed above. This time, however, the mechanism is not one of the revelation type; hence, Nash equilibria do not become dominance equilibria.

The pioneering contribution in this direction is due to Groves and Ledyard (1977).

The Groves and Ledyard idea may be illustrated by a mechanism close to (but not identical with) theirs. Consider again a three-person economy with two goods (a private good X and public good Y), and constant returns in producing Y from

[71] Bowen (1943), Green, Kohlberg and Laffont (1976), Green and Laffont (1979, pp. 157–188). For private goods, see Roberts and Postlewaite (1976).

[72] Green and Laffont (1979, pp. 213–227).

[73] See, for instance, d'Aspremont and Gérard-Varet (1979a, 1979b) and Arrow (1977).

X, the input–output coefficient normalized to 1. Assume a differentiable (but not necessarily transferable) utility function, denoted for the ith agent by $u^i(x^i, y)$.

Let each agent's strategy domain be the set of reals. Denote the ith strategy domain by $\mathscr{M}^i$ (here $\mathscr{M}^i = R$, $i = 1,2,3$) and its elements by m_i. Write $m = (m_1, m_2, m_3)$, and $\mathscr{M} = M^1 \times M^2 \times M^3$. The outcome function is defined by the tax functions T^i, $i = 1,2,3$, and the public good function Y. Given the strategy triple $m = (m_1, m_2, m_3)$, agent i pays tax equal to $t^i = T^i(m)$, and the amount of public good produced is $y = Y(m)$. Now specify these functions as follows:

$$T^i(m) = m_i^2 + 2m_j m_k, \qquad i = 1,2,3, \quad i \neq j \neq k \neq i,$$

$$Y(m) = M^2,$$

where

$$M = m_1 + m_2 + m_3.$$

Note first that the balance condition is satisfied because, for every $m \in M$,

$$Y(m) = M^2 = \sum_{i=1}^{3} \left(m_i^2 + 2m_j m_k\right) = \sum_{i=1}^{3} T^i(m).$$

Now examine the first-order necessary interior Nash equilibrium condition,

$$\partial v^i / \partial m_i = 0, \qquad i = 1,2,3,$$

where v^i is the payoff (indirect utility) function defined by

$$v^i(m) = u^i\left(\omega_x^i - T^i(m), Y(m)\right).$$

Hence

$$0 = \partial v^i / \partial m_i = u_x^i \cdot \left(-T_i^i\right) + u_y^i \cdot Y_i, \qquad i = 1,2,3,$$

where

$$T_i^i = \partial T^i / \partial m_i, \qquad \partial Y_i = \partial Y / m_i.$$

Using the outcome function specified above, this yields

$$u_x^i \cdot (-2m_i) + u_y^i \cdot 2M = 0, \qquad i = 1,2,3,$$

and hence

$$u_y^i / u_x^i = m_i / M, \qquad i = 1,2,3.$$

Summing, we obtain the Samuelson condition

$$\sum_{i=1}^{3} u_y^i / u_x^i = \sum_{i=1}^{3} m_i / M = \sum_{i=1}^{3} m_i \Big/ M = M/M = 1.$$

Thus, subject to the verification of second-order condition, we see that – locally at least – a Nash equilibrium allocation is Pareto-optimal.

For a precise version the reader is referred to Groves and Ledyard (1977, p. 796, esp. (4.3)–(4.4)), of which earlier version was circulated in 1974.

In a subsequent paper Groves and Ledyard (1980) (an early version was circulated in 1975) gave sufficient conditions for the existence of Nash equilibria in their model. As they point out, these conditions are slightly stronger than those required to prove the existence of Lindahl equilibria. Furthermore, the taxes may leave the consumers worse off than they had been with their initial endowments. I.e. the Groves–Ledyard mechanism is not *individually rational.*

6. Implementing the Lindahl correspondence

One is thus led to ask whether there exist alternative mechanisms whose Nash equilibrium allocations are not only Pareto-optimal but also individually rational. Also, it is of course desirable that equilibria exist for a wide class of economic environments.

All of these desiderata would be fulfilled if it were possible to design a mechanism whose Nash equilibria generate Lindahl allocations. We know already that this cannot be accomplished by a revelation game that is "natural for" (or "compatible with") the Lindahl correspondence. But there do exist mechanisms whose equilibrium allocations are precisely those of the Lindahl solution. Before describing them, a brief digression concerning *feasibility.*

In the public goods economies described above, there are two feasibility conditions: (a) "aggregate", $y = \sum_{i=1}^{n} t^i$, and (b) "individual", $t^i \leqq \omega_x^i$, for each $i = 1,\dots, n$. Translated into properties of outcome functions, these become (a′) $Y(s) = \sum_{i=1}^{n} T^i(s)$ for all s in S, and (b′) $T^i(s) \leqq \omega_x^i$ for all i in $N = \{1,\dots, n\}$ and all s in S (where $S = S^1 \times \cdots \times S^n$ is the Cartesian product of individual strategy domains). Condition (a′) is referred to as *balance*, and condition (b′) as *individual feasibility.*

Consider now the conventional scenario with the Lindahl analogue of a Walrasian auctioneer (Section 2). The auctioneer announces an n-tuple $p = (p_1,\dots, p_n) \gg 0$, $\sum_{i=1}^{n} p_i = 1$, of personalized prices; for each $i = 1,\dots, n$, the ith agent responds with the desired net trade commodity bundle (x^i, y^i) such that

$$x^i + p_i y^i \leqq 0,$$

and (x^i, y^i) is individually feasible.[74] In the usual interpretation, each agent treats the price parametrically and acts as if expecting to receive the requested bundle. Such a situation may be formalized as what has been called a *quasi-game*.[75] In a quasi-game there are two types of participants – not only the players (here, the n economic agents), but also non-players (here, the auctioneer). Each participant has a strategy domain. The ith agent's domain may be the set of individually feasible trades $Z^i = \{z^i : z^i \geqq -\omega^i\}$, and the auctioneer's domain may be the price space $P = \{(p_1, \dots, p_n) \in R^n_{++} : \sum_{i=1}^n p_i = 1\}$. Thus, in conventional notation the $(n+1)$-tuple of strategies is $S = (S^1, \dots, S^n, S^{n+1}) = (z^1, \dots, z^n, p)$. Suppose that the outcome function for the ith agent is given by

$$\begin{aligned} h^i(s) &= z^i \quad \text{if} \quad (1, p^i)\cdot z^i \leqq 0, \quad i = 1, \dots, n, \\ &= 0 \quad \text{otherwise,} \end{aligned}$$

and the auctioneer's outcome function by

$$h^{n+1}(s) = \left(\sum_{j \in N'} x^j, - \sum_{j \in N'} x^j \right),$$

where j ranges over the set N' of those agents j for whom $(1, p^j)\cdot z^j \leqq 0$. (Thus the auctioneer is supplying $y = \sum_{j \in N'} x^j$ in exchange for the tax revenue of $x = \sum_{j \in N'} x^j$. In effect, the auctioneer is also the producer or, at least, the supplier of the public good or service.)

What we have described so far is not yet a quasi-game, but only a mechanism (outcome function) which is individually feasible but not balanced[76] and involving a non-player participant. To define a quasi-game we must specify the participants' behavior rules. For each player, as usual, we define a payoff (indirect utility) function,

$$v^i(s) = u^i(h^i(s)), \qquad i = 1, \dots, n,$$

where u^i is the utility function (assumed to be strictly increasing) of the ith agent in terms of net trades. The auctioneer's payoff function may be defined as

$$v^{n+1}(s) = \left(- \max_{i,k \in N'} |y_i - y_k| \right) \cdot \max_i p_i,$$

where $p_i > 0$, $\forall i$, and $\sum_{i=1}^n p_i = 1$.

[74] I.e. $(x^i, y^i) \geqq -\omega^i \equiv -(\omega^i_x, 0)$.
[75] Hurwicz (1979b).
[76] Since it is not in general the case that $y^k = \sum_{j \in N'} x^j$ for all k such that $x^k + p_k y^k \leqq 0$.

A *Nash equilibrium* s^* of this quasi-game is defined, as usual, by

$$v^r(s^*) \geqq v^r(r, s^r/s^{*})^{r(}) \quad \text{for all} \quad s^r \in S^r, \qquad r = 1,\dots,n, n+1.^{77}$$

It is clear that the Nash equilibrium s^* for the outcome and payoff functions specified above is characterized by: (1) $y^{*1} = \cdots = y^{*n}$ ($= y^*$ say) since otherwise v^{n+1} is not maximized with respect to $s^{n+1} = p$; (2) $x^{*i} + p_i^* y^* = 0$ for all $i = 1,\dots,n$; and (3) $u^i(x^{*i}, y^*) \geqq u^i(x^i, y^i)$ for any (x^i, y^i) satisfying the budget constraint $x^i + p_i^* y^i \leqq 0$, since otherwise v^i is not being maximized with respect to $s^i = (x^i, y^i)$. Note also that the last condition implies that, at a Nash equilibrium, prices are treated parametrically.

Thus Nash equilibrium for this quasi-game yields a Lindahl equilibrium. But there are two features considered unsatisfactory by some. The first objection is to the introduction of a non-player participant, the auctioneer. This participant has a strategy variable but his payoff function is artificial – corresponding to the rule that the price vector should be chosen so as to equalize the agents' demands for public service. But it is not obvious how seriously to take this objection, since the role of the auctioneer could be programmed for a computer.

The second objection is to the lack of balance in the outcome function. When the agents' demands $y^1,\dots, y^n$ for public good are not all equal, the actual supply y (here equal to $\sum_{j \in N'} x^j$) must be different from y^i for some i. The allocation $z^i = (x^i, y^i) = h(s)$ specified by the outcome function is therefore different from the allocation that would in fact be made, say $\tilde{z}^i = (x^i, \sum_{j \in N'} x^j)$ if $x^i + p_i y^i \leqq 0$ or $\tilde{z} = (0,0)$, otherwise. One would, in effect be assuming that the agents are either acting in ignorance of what the actual outcome would be, or willing to act on an "as if" basis. Such lack of realism can only be avoided by constructing a game that is balanced.

Thus it becomes of interest to know that a balanced outcome function can be constructed to implement the Lindahl correspondence. Indeed one need not use a separate auctioneer or any other non-player participant.[78]

7. Balanced outcome functions without an auctioneer

For $n = 3$, and without satisfying the individual feasibility condition, such outcome functions were constructed by Hurwicz (1979a) and Walker (1981). In

[77] Hurwicz (1979b) gives a somewhat more general definition of Nash equilibrium in a quasi-game.

[78] In fact, the issue of constructing balanced outcome functions implementing a Pareto-optimal individually rational social choice rule without an auctioneer arose first in the context of implementing the Walrasian correspondence and was accomplished for $n \geqq 3$, although without satisfying the individual feasibility condition, by Schmeidler (1976).

Hurwicz (1979a) one may think of agents as arranged in a circle, with each agent setting the price (acting in effect as an auctioneer) for his/her neighbors.[79]

At the same time each agent proposes the level of the public good. Thus agent i's message is of the form $m^i = (p_i, y_i)$, where y_i is the proposed level of the public good and $p_i \geqq 0$ will serve to determine the price to be paid by certain agents other than i. Specifically, the Lindahl price to be paid by agent j is[80]

$$R_j(m) = 1/n + p_{j+1} - p_{j+2},$$

where j is taken modulo n. (I.e., $n+1 \equiv_n 1$, $n+2 \equiv_n 2$, etc.) The outcome function is defined as follows:

$$Y(m) = \sum_{j=1}^{n} y_j/n,$$

and

$$X^i(m) = -R_i(m)\cdot Y(m) - p_i(y_i - y_{i+1})^2 + p_{i+1}(y_{i+1} - y_{i+2})^2, \quad i = 1,\dots,n.$$

As is necessary for the Lindahl prices in this normalized model, we have

$$\sum_{i=1}^{n} R_i(m) = 1 \quad \text{for all } m,$$

and hence

$$\sum_{i=1}^{n} X^i(m) = -Y(m)\sum_{j=1}^{n} R_i(m) - \sum_{i=1}^{n} p_i(y_i - y_{i+1})^2 + \sum_{i=1}^{n} p_{i+1}(y_{i+1} - y_{i+2})^2.$$

Now the last two terms cancel out and so

$$\sum_{i=1}^{n} X^i(m) = -Y(m)\cdot R_i(m) = -Y(m).$$

Hence the mechanism is balanced.[81]

[79] Thus every agent is forced to treat the price parametrically because, at a Nash equilibrium, the other players' strategies are taken as given.

[80] $m = (m^1,\dots,m^n)$, $m^i = (p_i, y_i)$, $i = 1,\dots,n$. All summations, unless otherwise indicated, are from 1 to n.

[81] The balance is essentially due to the presence of the term $p_{i+1}(y_{i+1} - y_{i+2})^2$ in $X^i(m)$. Note that this term cannot be influenced by agent i when $n \geqq 3$. (For $n = 2$, $y_{i+2} = y_i$.)

On the other hand, it is not the case that $(X^i(m), Y(m)) \geqq 0$ for all m. Hence the mechanism does not satisfy the condition of individual feasibility.[82]

It turns out that, at a Nash equilibrium $m^* = \{(y_i^*, p_i^*)\}_{i=1}^n$, we have, for every i, $y_i^* = y_{i+1}^*$ or $p_i^* = 0$, and hence

$$X^i(m^*) = -R_i(m^*)Y(m^*),$$

which is the Lindahl budget equation.

It was then shown that, for $n \geqq 3$, the mechanism so constructed (fully) implements the Lindahl correspondence.

A simpler mechanism, using a smaller message space,[83] was constructed in Walker (1981). Here each player has a one-dimensional strategy (message) space $M_i = R$. The outcome function is given [with $m = (m_1, \ldots, m_n)$] by

$$Y(m) = \sum m_i,$$

and (in terms of net trades)

$$X^i(m) = -Y(m) \cdot q^i(m),$$

where

$$q^i(m) = 1/n + m_{i+2} - m_{i+1}$$

is the ith agent's Lindahl price [with $\sum q^i(m) = 1$], again outside the control of the ith agent. The balance property follows from $\sum X^i(m) = -Y(m)\sum q^i(m) = -Y(m)$ for all m. Again, however, individual feasibility may be violated.[84] For $n \geqq 3$, this mechanism is shown to (strongly) implement the Lindahl correspondence. It is obvious that[85] it does so with a message space of minimal dimension.[86]

Neither of the preceding two mechanisms implements the Lindahl correspondence when there are only two agents, and both violate the condition of individual feasibility. But we shall now see that either one of these defects may be avoided.

[82] In Hurwicz (1979a) the preference relation $\succcurlyeq_i$ was extended to all R^2, so formally the mechanism was well-defined, and individually feasible in terms of *extended* preferences. But there is no realistic reason why an agent would prefer one infeasible bundle to another.

[83] And having other advantages, see Walker (1981, p. 66, footnote 2). In addition, Walker shows how to generalize this mechanism to many agents and goods and more general production relations (ibid., pp. 68–69).

[84] See Walker (1981, p. 67, footnote 3) where, however, the point is made that an *interior* equilibrium has a neighborhood on which feasibility is assured. The same applies to Groves and Ledyard (1977) as well as Schmeidler (1976, 1980) and Hurwicz (1979a).

[85] For a sufficiently rich class of environments.

[86] Cf. Reichelstein (1983).

For an economy with two agents, a mechanism has been constructed by Miura (1982).[87] Here again $m^i = (p_i, y_i)$, $i = 1,\dots,n$, and the outcome function is given by[88]

$$Y(m) = \sum y_j/n, \quad X^i(m) = -\left(p_i/\sum p_j\right)Y(m) \quad \text{if} \quad \prod p_j = 1,$$

while

$$Y(m) = p_j - 1, \quad X^i(m) = -\left(p_i/\sum p_j\right)Y(m) \quad \text{if} \quad \prod p_j \neq 1.$$

This outcome function is balanced under both regimes, but – like Hurwicz (1979c) – it is discontinuous and violates the individual feasibility condition; also, it uses a message space bigger than Walker's (1981).[89] It does implement the Lindahl correspondence for $n = 2$.[90]

Completely feasible implementation. The issue of implementation satisfying both the balance and individual feasibility conditions (we call this *completely feasible implementation*) is treated in Hurwicz, Maskin and Postlewaite (1980). It is shown there that, for $n \geqq 3$, the constrained[91] Lindahl correspondence can be Nash-

[87] This is a modification of the mechanism in Hurwicz (1979c) which Miura (1982) showed to contain an error. Hurwicz (1979) used the condition $\Sigma_{j=1}^n p_j = 1$ rather than $\Pi_{j=1}^n p_j = 1$ to distinguish between two regimes.

[88] All summations and products are from 1 to n.

[89] By an argument analogous to Hurwicz (1976, appendix 1) one can show that no smooth balanced Nash implementation of any Pareto-optimal correspondence is possible for $n = 2$ with a 2-dimensional message space.

[90] Note that the previously discussed mechanisms [Hurwicz (1979) and Walker (1981)], for $n \geqq 3$, are not only continuous but even smooth. For $n = 2$, it appears that the Lindahl correspondence cannot be Nash-implemented smoothly by a balanced outcome function [using an argument analogous to Reichelstein (1984)].

[91] In the setting of footnote 30, a *constrained Lindahl allocation* $L_c(e)$ for environment e, denoted by $L_c(e)$, differs from an ordinary Lindahl allocation $L(e)$ in that condition (2) is replaced by the following (2′): for some $(p_1,\dots,p_n)$ and all $i \in N$, if $(x^i, y) \in C^i$, $x^i + yp_i \leqq \omega_x^i + \omega_y^i p_i$, and $x^i \leqq \Sigma_{i \in N}\omega_x^i$, then $(\bar{x}^i, \bar{y})\ R^i(e)\ (x^i, y)$.

Analogously, an allocation $(\bar{x}_i)_{i \in N}$, $\bar{x}_i \in R_+^l$ and a price vector p constitute a (pure exchange) *constrained Walrasian equilibrium* if

(i) $i \in N$, $p \cdot \bar{x}_i = p \cdot \omega_i$;

(ii) $i \in N$, $x_i R_i x$ for all $x \leqq \omega$, $\omega \equiv \Sigma\omega_i$, such that $p \cdot x \leqq p \cdot \omega_i$;

(iii) $\Sigma_{i \in N}\bar{x}_i = \omega$.

Note that $\bar{x}_i \in R_+^l$ represents total holdings (*not* the net trade) of the ith agent.

It is the condition "for all $x \leqq \omega$" that distinguishes this from the ordinary Walrasian equilibrium, since here the ith agent maximizes satisfaction subject not only to the individual budget constraint but also constrained by the aggregate availability of resources in the whole economy. (Similar remark applies to the condition $x^i \leqq \Sigma\omega_x^i$ in the definition of constrained Lindahl equilibrium.)

implemented by characteristic profile strategies over a class of environments E for which Lindahl allocations leave everyone with some private goods, i.e., such that for each e in E,

$$\omega_x^i + L_x^i(e) \geq 0,$$

where $L_x^i(e)$ is the net Lindahl increment of private good X obtained by agent i in the environment e.

In Hurwicz, Maskin, and Postlewaite (1980), a generic element of agent i's strategy space is of the form

$$s_i = \left(e_i^1, \dots, e_i^n, y_i\right), \qquad e_i^j = \left(w_i^j, R_i^j\right),$$

where y_i is i's proposal for the level of public goods, and e_i^j is agent i's statement concerning agent j's characteristic; w_i^j and R_i^j are, respectively, j's X-endowment and j's preference relation according to i. It is postulated that no agent can exaggerate his own endowment, i.e., $w_i^j \leqq \omega_i$ for all i. The outcome function is formulated in such a way that only truthful unanimity as to endowments can prevail at a Nash equilibrium. Once such unanimity as to endowments has taken place, a game of type considered in Maskin's Theorem 5 guarantees the Nash implementation of the (constrained) Lindahl correspondence through the use of preference profiles as strategies.

The result obtained [Hurwicz, Maskin and Postlewaite (1980, theorem VII)] applies to a class of performance correspondences broader than (constrained) Lindahl; it is sufficient that they be individually rational and monotone;[92] monotonicity is also necessary by Maskin's (1977) Theorem 2. This result presupposes that there are at least three agents, that the endowments are semi-positive, and preferences strictly increasing.

Every Walrasian equilibrium is a constrained Walrasian equilibrium. Every *interior* constrained Walrasian equilibrium is a Walrasian equilibrium.

When non-interior equilibrium allocations can occur, the (ordinary) Walrasian and Lindahl correspondences are not monotone, hence [by Theorem 2 in Maskin (1977)] are not Nash-implementable (see next page). The smallest monotone correspondences containing these are the corresponding constrained correspondences. Hence they are the smallest supercorrespondences that have a chance of being implementable.

[92]A performance correspondence $F: \mathscr{R} \Rightarrow A$ defined on the family $\mathscr{R}$ of preference profiles into the feasible set A is said to be *monotone* if, for any a in A, and any two profiles $\boldsymbol{R}, \boldsymbol{R}'$, the following holds: if (1) $a \in F(\boldsymbol{R})$, and (2) aR_ib implies $aR_i'b$ for all $i \in \{1,\dots,n\}$ and all $b \in A$, then $a \in F(\boldsymbol{R}')$. I.e. if $\underline{a}$ is F-desirable for profile $\boldsymbol{R}$, and another profile $\boldsymbol{R}'$ is at least as favorable to $\underline{a}$ as $\boldsymbol{R}$ was, then $\underline{a}$ remains F-desirable for profile $\boldsymbol{R}'$. [Here $\boldsymbol{R} = (R_1,\dots,R_n)$, $\boldsymbol{R}' = (R_1',\dots,R_n')$. "$\underline{a}$ is F-desirable for environment $\underline{e}$" simply means $a \in F(e)$. Maskin (1977) uses the term "F-optimal". "At least as favorable" here permits replacing preference by indifference.]

It may be noted that the "no veto power"[93] condition used by Maskin in theorem 5 is not necessarily satisfied by the Lindahl or other performance correspondences to which Theorem VII of Hurwicz, Maskin, and Postlewaite (1980) applies. However, it is shown that, under the assumptions made, the "no-veto power" condition can be dispensed with.

Unfortunately, profile-using mechanisms require huge message spaces and are discontinuous. However, it may be possible to construct continuous mechanisms using smaller spaces, in a manner analogous to that used in Postlewaite and Wettstein (1983) for the implementation of Walrasian correspondence, but this question is still open. So is the problem of designing a feasible mechanism to implement the (constrained) Lindahl correspondence when there are only two agents.

8. Implications of Nash-implementability

We have already mentioned Maskin's result that only monotone performance correspondences are Nash-implementable. An example due to Postlewaite[94] shows that the Walrasian correspondence containing boundary allocations is not monotone; an analogous example could be constructed for the Lindahl correspondence. It is for this reason that the Nash-implementability results are for "constrained" Walrasian (respectively Lindahl) correspondences; these constrained correspondences are the smallest monotone ones containing the Walrasian (respectively Lindahl) correspondences.

Now it turns out that, in conjunction with other frequently made assumptions, monotonicity has very strong implications. In particular, suppose that a correspondence F is not only monotone, but also Pareto-optimal, individually rational, and continuous, over a sufficiently rich class of preferences. If we are dealing with pure exchange economies, it follows that

$$F(e) \supseteqq W_c(e) \quad \text{for all } e \text{ in } E, \tag{8.1}$$

where W_c is the constrained[95] Walrasian correspondence and $E \supseteq E_{L_c}$, the class of economies specified in Hurwicz (1979b).[96] Similarly, if we are dealing with a

[93]A performance correspondence $F: \mathscr{R} \Rightarrow A$ is said to have the "*no veto power*" property when the following is true: if, for any a in A, and $\boldsymbol{R}$ in $\mathscr{R}$, and for some i in $\{1, \ldots, n\}$, we have aR_jb for all $b \in A$ and all $j \neq i$, then $a \in F(\boldsymbol{R})$. I.e. if the prevailing preferences are such that a is the most preferred alternative either for all players or for all players but one, then a is F-desirable.

[94]Described in Hurwicz, Maskin and Postlewaite (1980).

[95]See footnote 91.

[96]For $n \geqq 3$ this can be proved [see Hurwicz, Maskin and Postlewaite (1980)] by using Theorem 1 in Hurwicz (1979b) and Theorem 2 in Maskin (1977). However, a direct (unpublished) proof is available, valid for all $n \geqq 1$.

public goods economy,

$$F(e) \supseteqq L_c(e) \quad \text{for all } e \text{ in } E,$$

where L_c is the constrained Lindahl correspondence.

Now consider an arbitrary correspondence F which is continuous, Pareto-optimal, individually rational, and Nash-implementable over $E \supseteq E_{L_c}$. Since Nash-implementability implies monotonicity, it follows that $F \supseteqq W_c$ if we are in a private goods, pure exchange economy, or that $F \supseteqq L_c$ in a public goods economy. In other words a continuous, Pareto-optimal, individually rational correspondence, which does not contain the Walrasian (respectively Lindahl) performance correspondence, is not Nash-implementable. In particular, if F is singleton-valued, continuous, Pareto-optimal, and individually rational, it is Nash-implementable only if it is constrained Walrasian.[97]

A partial converse is obtained under additional (convexity or starlike) restrictions on the outcome function [Hurwicz (1979c) and Schmeidler (1982)].

If, in the above assumptions, *fairness* (in the sense of absence of envy) replaces individual rationality, it has been shown by Thomson (1979) that analogous conclusions obtain for a pure exchange economy, namely with formula (8.1) being replaced by

$$F(e) \supseteqq W_{c,I}(e),$$

where $W_{c,I}(e)$ is the constrained Walrasian allocation that would follow equal distribution of endowments.

Hence, if, in a private goods, pure exchange economy, F is continuous, Pareto-optimal, and envy-free, it is Nash-implementable only if it contains $W_{c,I}$. Analogous results were obtained by Thomson for other concepts of fairness, e.g. that of egalitarian equivalent [Pazner and Schmeidler (1978)]. Partial converses were also obtained.

9. Private goods, pure exchange economies

The preceding exposition has been focused on the public goods problem. But there are analogous results for private goods economies. The earliest balanced outcome function Nash-implementing the Walrasian correspondence for $n \geq 3$ is due to Schmeidler (1976). A slightly modified form [Hurwicz (1979c)] works for all $n \geqq 2$. Here the ith strategy $m^i = (p_i, y_i)$, where both components are of dimension $l-1$, is an economy with goods X and Y [X one-dimensional, Y $(l-1)$-dimensional]. p_i is the price of Y while X is the numeraire.

[97] For a related result in "large" economies, see Theorem 5 in Hammond (1979).

Given an n-tuple $m=(m^1,\dots,m^n)$ of strategies, the set N of agents is partitioned so that (1) if agent i has proposed a price vector p_i that has not been proposed by anyone else, then i belongs to the subset T_0 of N, and only such "loners" belong to T_0; (2) if a price vector q has been proposed by two or more members of N, then all those proposing q belong to the subset $T(q)$.

Let $q^1,\dots,q^k$ be the complete list of price vectors each of which has been proposed by at least two agents, and write $T(q^r)=T_r$. Thus N is partitioned into subsets $T_0, T_1,\dots,T_k$, with some of these possibly empty. The outcome function is written

$$h(m)=\{X^i(m),Y^i(m)\}_{i=1}^{m}.$$

If $T_0 \neq N$ and agent i belongs to the subset T_r, $r\in\{1,\dots,k\}$, then

$$Y^i(m)=y_i-\Big(\sum_{j\in T_r} y_j\Big)\Big/\#T_r,$$

$$X^i(m)=-q^r\cdot Y^i(m).$$

(This is precisely the Schmeidler rule.)

If T_0 is non-empty and i belongs to T_0, then

$$Y^i(m)=p_i-P,\qquad X^i(m)=-P\cdot Y^i(m),$$

where

$$P=\Big(\sum_{j\in T_0} p_j\Big)\Big/\#T_0.$$

(This rule is different from Schmeidler's rule for T_0.)

This outcome function is balanced but outcomes need not be non-negative; i.e., individual feasibility is not guaranteed. But with regard to extended preference relations, it (fully) Nash-implements the Walrasian correspondence for all $n\geqq 2$.

The preceding outçome function is discontinuous. A smooth balanced outcome function, also Nash-implementing the Walrasian correspondence (but again only for extended preferences, hence violating individual feasibility) is given, for $m=(m^1,\dots,m^n)$, $m^i=(p_i,y_i)$, p_i, y_i both $(l-1)$-dimensional vectors by $h(m) = \{X^i(m)=(X^i(m), Y^i(m))\}_{i=1}^n$, as follows: for each $i=1,\dots,n$,

$$Y^i(m)=y_i-y_{-i},$$

where

$$y_{-i}=\Big(\sum_{j\neq i} y_j\Big)\Big/(n-1),$$

$$X^i(m)=-p_{-i}\cdot Y^i(m)-L^i(m)+S^i(m),$$

where

$$p_{-i} = \Big(\sum_{j\neq i} p_j\Big)\Big/(n-1),$$

$$L^i(m) = (p_i - p_{-i})\cdot(p_i - p_{-i}),$$

and $S^i(m)$, which does not depend on m^i, is such that $\sum_{i=1}^n X^i(m) = 0$ for all m. Since $\sum_{i=1}^n Y^i(m) = 0$ for all m, the outcome function is balanced. The "penalty" term $L^i(m)$ provides an incentive to equalize all price proposals, and the "budget" term $-p_{-i}\cdot Y^i(m)$ prevents all agents from being their own price-setters.[98]

Completely feasible implementation. On the other hand, it is also possible to Nash-implement the Walrasian correspondence by a balanced outcome function without violating the individual feasibility condition, in a manner analogous to that outlined above for the Lindahl correspondence. In fact, a more general result is available. Assume $n \geqq 3$, and consider a performance function[99] f (on a class E of pure exchange economies) which is individually rational and Nash-implementable by an outcome function g_v on strategy domain $D = D^1 \times \cdots \times D^n$ when v is a *known* endowment profile.

To implement f when the endowments are not known to the designer we give the ith agent the strategy domain S^i whose generic element is of the form

$$s_i = \left(w_i^1, \ldots, w_i^n, d_i\right),$$

where w_i^j is i's statement concerning the endowment of agent j,[100] and d_i an element of D^i. The outcome function on $S = S_1 \times \cdots \times S_n$ (for the game in which initial endowments are *not* known) is so designed that a Nash equilibrium can occur only when, for all i, $(w_i^1, \ldots, w_i^n) = (\dot{\omega}_1, \ldots, \dot{\omega}_n)$ = the true endowment profile. When such unanimity occurs, what remains is in effect the game form (D, g_v) implementing f on the assumption that the unanimously agreed endowment profile v is the correct one.

[98]An earlier version of Hurwicz (1979a) contained a "circular" variant, with $X^i(m) = -Y^i(m)\cdot p_{i-1} - L^i$ + a balancing term independent of r^i. Here, as in the above public goods mechanism, each agent is his neighbor's price-setter.

[99]With minor modifications of the outcome functions, these results can be extended to correspondences.

[100]It is assumed that $w^i \leqq \dot{\omega}_i$ for all v, i.e. one cannot exaggerate one's endowment. See Hurwicz, Maskin and Postlewaite (1980).

Since f is assumed Nash-implementable for known endowments, it must [by Maskin (1977, theorem 2)] be monotone. We have also assumed $n \geqq 3$. If f has the "no-veto property", then Maskin's Theorems 4 and 5 provide us with a game form with a generic term of strategy domain D^i of the form

$$d_i = (R_i^1, \ldots, R_i^n),$$

where R_i^j is the statement by i concerning the preference relation of agent j.

Maskin's Theorems 4 and 5 do more than construct a game form for a particular performance correspondence; these theorems constitute a recipe for constructing game forms Nash-implementing a large class of correspondences when there are at least three players ($n \geqq 3$). Let $F: E \Rightarrow A$ be a correspondence to be implemented in an economy where A is the set of feasible outcomes, $R = R_1 \times \cdots \times R_n$ is a class of preference profiles, and

$$E_A = \{e : e = (A; R_1, \ldots, R_n), (R_1, \ldots, R_n) \in R_1 \times \cdots \times R_n\}.$$

For each i, R_i is a preference preordering (a total, transitive and reflexive binary relation) on A. It is assumed that A and R are a priori known to the designer.

Agent i's strategy space is

$$S_F^i = \{(R_1, \ldots, R_n, a) : (R_1, \ldots, R_n) \in R,\ a \in F(A; R_1, \ldots, R_n)\}.$$

A generic element of S^i is of the form

$$s_i = (R_i^1, \ldots, R_i^n, a),$$

where R_i^j is i's statement about j's preference relation.

Note that when F is singleton-valued (i.e. a function), the a-component of s_i can be omitted. Theorem 4 states that an outcome function $h: S \to A$, $S = S^1 \times \cdots \times S^n$, fully implements F if it has the following three properties:

(i) If there is unanimity, i.e. if, for some $(\bar{R}_1, \ldots, \bar{R}_n) \in R$ and $\bar{a} \in F(\bar{R}_1, \ldots, \bar{R}_n)$, all agents' strategies satisfy $\bar{s}_1 = \cdots = \bar{s}_n = (\bar{R}_1, \ldots, \bar{R}_n, \bar{a})$, then[101]

$$h(\bar{s}_1, \ldots, \bar{s}_n) = \bar{a}.$$

(ii) For any agent $i \in \{1, \ldots, n\}$, let $\bar{s}_1 = \cdots = \bar{s}_{i-1} = \bar{s}_{i+1} = \cdots \bar{s}_n = (\bar{R}_1, \ldots, \bar{R}_n, \bar{a})$, with $\bar{a} \in F(\bar{R}_1, \ldots, \bar{R}_n)$; then[102]

$$\{b \in A : b = h(\bar{s}_1, \ldots, \bar{s}_{i-1}, s_i, \bar{s}_{i+1} = \cdots = \bar{s}_n), s_i \in S^i\}$$
$$= \{c \in A : \bar{a}\bar{R}_i c\}$$
$$= \text{the lower contour set of } \bar{a} \text{ under } \bar{R}_i.$$

[101] Note that when F is a function this relation becomes $h(\bar{s}) = F(\bar{s})$.

[102] I.e. when all agents but i are unanimous, every outcome in A can be reached by i through unilateral choice.

(iii) For $\bar{s} \in S$, let there exist $i \in \{1,\dots,n\}$ such that "$\bar{s}_1 = \cdots = \bar{s}_{i-1} = \bar{s}_{i+1} = \bar{s}_n$" is false; then[103]

$$\{b \in A : b = h(\bar{s}_1,\dots,\bar{s}_{i-1},\bar{s}_i,\bar{s}_{i+1},\dots,\bar{s}_n),\, s_i \in S^i\} = A.$$

Maskin's Theorem 5 shows by construction that when $n \geqq 3$, and F is monotone and has the "no-veto power" property, then there exists an outcome function h satisfying the above conditions (i), (ii), (iii). [See, however, Williams (1984a, 1984b) and Saijo (1984).]

However, the strategy domain used in the above example is extremely large, and the outcome function discontinuous. For the special case of a Walrasian correspondence, $n \geqq 3$, Postlewaite and Wettstein (1983) have designed a balanced *continuous* outcome function, with a generic element of D^i of the form

$$d_i = (z^i, p^i, r^i),$$

where z^i and p^i are, respectively, a net trade vector and a price vector announced by individual i, and r^i a positive real number. Thus the dimension of D^i is $1+2l$, where l is the total number of goods.

Reichelstein (1982) has constructed, for $n \geqq 3$, smooth mechanisms Nash-implementing the Walrasian correspondence with strategic domains of smaller dimensions, but these do not satisfy the individual feasibility condition. It is not known at present what the minimal required dimension is when we insist on individual feasibility as well as balance and continuity or smoothness. [But see Williams (1984c).]

10. Informational aspects of Nash-implementability

An exciting aspect of recent research is the convergence of informational and incentive aspects of economic mechanisms. One example of this is the investigation of minimal dimensional requirements[104] on the strategy spaces of mechanisms implementing the Pareto (and, in particular, Walras or Lindahl) correspondence. Another is a study of the relationship between message mechanisms and game forms giving rise to Nash equilibrium outcomes.[105]

It is clear that such a game form may be viewed as arising from an adjustment process or a message mechanism. Given a game form (S, h), let $\nu_{S,h}$ be its Nash equilibrium correspondence over E; i.e. for each e in E,

$$\nu_{S,h}(e) = \{s \in S : s \text{ is a Nash equilibrium for the game } (S; h, e)\},$$

[103] I.e. when there is no unanimity among agents other than i, agent i can reach every feasible point by unilateral choice.

[104] Reichelstein (1982) and Hurwicz (1976).

[105] Williams (1984b).

where (h, e) is the payoff function defined by the composition of the outcome function h with the preferences in e. E.g. if $e = (e^1, \ldots, e^n)$, $e^i = (u^i, \ldots)$, where u^i is the ith utility function, then the ith payoff function is $v^i = u^i \circ h$.

Furthermore the resulting process is privacy-preserving. This is seen most easily when the preferences are represented by concave differentiable functions u^i and the outcome function h is also concave differentiable. Then the ith payoff function is $v^i = u^i \circ h$. At a Nash equilibrium, $\partial v^i / \partial s_i = 0$ for all i. More explicitly, in an l-dimensional commodity space, and with a k_i-dimensional strategy vector s_i, the condition becomes

$$\sum_{j=1}^{l} \left(\partial u^i / \partial x_j^i\right)\left(\partial h_j^i / \partial s_{i, r_i}\right) = 0, \qquad r_i = 1, \ldots, k_i.$$

It is clear that each agent can verify this condition knowing only the values of the strategy variables s and his/her own utility function. So the process is privacy-preserving.

Given a game form (S, h) which Nash-implements the correspondence F, we can easily construct a privacy-preserving message mechanism (S, ν_{Sh}, h) which realizes F. But the converse problem is much more difficult: given a privacy-preserving message mechanism $(\mathcal{M}, \mu, h)$ realizing F, to construct a game form (S, h) which Nash-implements F. Sufficient conditions for such construction are given by Williams (1984b).

In particular, let $(\mathcal{M}, \mu, h)$ be the natural direct revelation mechanism for a social choice *function* $f: E \to Z$. Thus $\mathcal{M} = \mathcal{M}^1 \times \cdots \times \mathcal{M}^n = E^1 \times \cdots \times E^n = E$, $h = f$, and μ is given by

$$m_i = e^i \quad \text{for all } i.$$

Then the Williams construction (which, however, covers a much broader class of mechanisms) yields a Maskin-type game form, with $S = E$.

The present paper fails to cover a number of important topics: the Harsanyi–Bayes type mechanism, the dynamics of allocation processes (e.g. Malinvaud, Drèze and de la Vallée Poussin), performance of mechanisms in large economies, and many others.

Several excellent surveys cover some of the topics neglected here as well as many that are discussed in the present essay [e.g. Dasgupta, Hammond and Maskin (1979), Groves (1982), Laffont and Maskin (1982), Postlewaite (1983)].

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