III. Magnetic and electric confinement of quarks *

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I.

The Nielsen–Olesen interpretation [1] of the string model requires the quarks to be magnetic monopoles. In the case of an Abelian gauge theory, and at least at the level of classical theory, this seems to be a consistent picture. One can combine the London theory of superconductivity and the Dirac theory of monopoles to describe the motion of dual strings [2]. It is a phenomenological step backward from a search for the dynamical origin of strings, but I am interested in the possibility, or existence, of just such a phenomenological framework in hadron physics. If we can indeed find a phenomenological framework, probably we can also find a microscopic and dynamical model from which it follows.

We are faced with two problems in our task. One is a non-Abelian generalization, and the other is quantization. The non-Abelian extention is a more pressing need in order to decide whether or not the string picture makes sense in hadron physics. Perhaps the quantum aspects become more essential in non-Abelian theories, and both must go hand in hand. The main question, at any rate, is how to construct mesons and baryons in the color SU(3) theory n, extended to include magnetic monopoles [3]. It is not obvious to me, however, that a non-Abelian monopole theory is possible without breaking the symmetry. This is because a string à la Nielsen–Olesen, or a monopole à la 't Hooft [4], is a classical particular solution to a gauge theory. From the phenomenological viewpoint, we attach a color octet magnetic flux density ρ^i at each point on the string. Since it should be parameter-independent, we may regard it as a field $\rho^i(x)$, and its covariant derivatives within the world sheet must be zero: $\mathrm{D}_\tau^{(\mathrm{A})}\rho^i = \mathrm{D}_\sigma^{(\mathrm{A})}\rho^i = 0$. Integrability then requires $[\mathrm{D}_\tau^{(\mathrm{A})}, \mathrm{D}_\sigma^{(\mathrm{A})}]\rho^i \sim f^{ijk}F^j_{\tau\sigma}\rho^k = 0$. Thus ρ^i and $F^i_{\tau\sigma} = F^i_{\mu\nu}\dot{x}_\mu x'_\nu$ must be "parallel". (For more details, see Eguchi [3].) This is a severe constraint; the only solution seems to be a trivial one (a no-go theorem?) which, under a suitable gauge, makes all components of ρ^i, $F^i_{\mu\nu}$, A^i_μ zero except for $i = 3$ and 8. We have then two uncoupled Abelian gauge fields corresponding to color isospin and hypercharge. Quark confinement certainly occurs, but the quarks are not necessarily in a color singlet state. This is only a classical solution. In quantum theory one cannot put certain components of the field equal to zero; symmetry-restoring fluctuations will be present, and might destroy the whole picture if the symmetry were to remain intact.

II.

The second problem I would like to discuss is the possibility of electric, rather than magnetic, confinement. Although it sounds like a matter of semantics, the mathematical formulations are different. Here I follow the elegant ideas due to P. Ramond. Most of what I say below seems to have been considered by Kalb and Ramond [5] in their published and unpublished work.

* Work supported in part by the National Science Foundation, Contract No. MPS74-08833.

Let us again start from an Abelian case. Consider a hierarchy of potentials A_μ, $A_{\mu\nu}$, $A_{\mu\nu\lambda}$, which are antisymmetric (except for the first) and undergo gauge transformations of the first kind:

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

$$A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$

$$A_{\mu\nu\lambda} \to A_{\mu\nu\lambda} + \sum_{\text{cycl}} \partial_\mu \Lambda_{\nu\lambda} \, .$$

The sources of the A's are point charge, loop (closed string) and closed shell, respectively. Denote their currents by j_μ, $j_{\mu\nu}$, $j_{\mu\nu\lambda}$, which satisfy $\partial_\mu j_\mu = \partial_\mu j_{\mu\nu} = \partial_\mu j_{\mu\nu\lambda} = 0$. These gauge transformations are related to an infinitesimal displacement forming a line $\partial_\mu \Lambda \, \mathrm{d}x_\mu$, a loop $\oint \Lambda_\mu \cdot \mathrm{d}x_\mu$, a shell $\oint\oint \Lambda_{\mu\nu} \, \mathrm{d}\Omega_{\mu\nu}$, by virtue of the Stokes' theorem. Therefore there exist a hierarchy of gauge invariant derivatives

$$\partial_\mu - gA_\mu , \qquad \frac{\delta}{\delta \Omega_{\mu\nu}} - gA_{\mu\nu} , \qquad \frac{\delta}{\delta \Omega_{\mu\nu\lambda}} - gA_{\mu\nu\lambda} \, .$$

They are to be applied to the wave functions $\psi(x)$, $\psi[x(\sigma)]$, $\psi[x(\sigma_1, \sigma_2)]$, for a point, a loop and a shell. The functional derivative $\delta/\delta\Omega_{\mu\nu}$ stands for the variation on $\psi[x(\sigma)]$ caused by an infinitesimal change of the string configuration which sweeps a surface element $\mathrm{d}\Omega_{\mu\nu}$, and so on. The gauge transformations of the first kind acting on them will be

$$\psi(x) \to \exp\{gi\Lambda(x)\} \, \psi(x)$$

$$\psi[x(\sigma)] \to \exp\{gi \int_\sigma \Lambda_\mu \, \mathrm{d}x_\mu\} \, \psi[x(\sigma)]$$

$$\psi[x(\sigma_1, \sigma_2)] \to \exp\{gi \iint \Lambda_{\mu\nu} \, \mathrm{d}\Omega_{\mu\nu}\} \, \psi[x(\sigma_1, \sigma_2)] \, .$$

Now the gauge invariant fields derived from the potentials are

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu\lambda} = \sum_{\text{cycl}} \partial_\mu A_{\nu\lambda}$$

$$F_{\mu\nu\lambda\rho} = \sum_{\text{cycl}} \partial_\mu A_{\nu\lambda\rho}$$

so that their respective Lagrangians are

$$-\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} + gA_\mu j_\mu$$

$$-\frac{1}{12m^2} F_{\mu\nu\lambda} F_{\mu\nu\lambda} + gA_{\mu\nu} j_{\mu\nu}$$

$$-\frac{1}{48m^4} F_{\mu\nu\lambda\rho} F_{\mu\nu\lambda\rho} + gA_{\mu\nu\lambda} j_{\mu\nu\lambda} \, .$$

The necessity of a dimensional constant m is somewhat surprising. The above construction may be extended to include open strings and shells. Since the boundary of a shell is a string, and so on, the continuity equations force these sources and fields to be coupled to each other.

From here on, let us concentrate on strings only. As has been shown by Cremmer and Scherk [6] and by Kalb and Ramond, the field $F_{\mu\nu\lambda}$ describes massless zero-helicity quanta in contrast to the usual Maxwell field $F_{\mu\nu}$. There is also a static (unquantized) part created by a source; it leads to an interaction between string elements similar to that between electric currents, albeit with an opposite sign. In other words, the strings behave like hydrodynamic vortices.

This is a great curiosity, a hitherto unknown long range force generated by (closed) strings (or vortices). Does such a phenomenon exist in nature?

Putting this intriguing question aside, let us give a mass to $A_{\mu\nu}$ and thereby break the gauge invariance (of the Ramond variety). Conceiveably it can be accomplished as a spontaneous breakdown, due to a superfluid of closed strings in the vacuum. (Consider the London-type ansatz $A_{\mu\nu} \propto j_{\mu\nu}$.)

We can then compute the interaction between open strings (as well as their self energies) through the massive Kalb–Ramond field. It turns out that the result is exactly the same as in the case of magnetic strings based on the Dirac–London theory. We call the present case an electric confinement because the electric field A_{0i} (though it is actually a potential) is static and non-zero, and there is no violation of P and T anywhere in the theory. The mathematical details are given in the Appendix.

When one seeks a non-Abelian version of the extended gauge principle, unfortunately one runs into trouble again for reasons similar to those encountered before *. But the gauge invariance in the Yang–Mills sense can be maintained by using covariant derivatives in defining the fields $F^i_{\mu\nu}$, $F^i_{\mu\nu\lambda}$, etc.

Appendix

The effective massive Lagrangian density for the Cremmer–Scherk–Kalb–Ramond field is

$$L = -\tfrac{1}{4}A_{\mu\nu}A_{\mu\nu} - \frac{1}{12m^2}F_{\mu\nu\lambda}F_{\mu\nu\lambda} + A_{\mu\nu}j_{\mu\nu}$$

$$F_{\mu\nu\lambda} = \sum_{\text{cycl}} \partial_\mu G_{\mu\lambda}$$

$$j_{\mu\nu}(x) = g\iint \frac{\partial(y_\mu, y_\nu)}{\partial(\tau,\sigma)}\ \delta^4(y-x)\,\mathrm{d}\tau\,\mathrm{d}\sigma\,.$$

With the notation $A_{\mu\nu} = (\boldsymbol{B}, \boldsymbol{E})$ in analogy with the electromagnetic field, we have

$$L = -\tfrac{1}{2}(\boldsymbol{B}^2 - \boldsymbol{E}^2) - \frac{1}{2m^2}\{(\nabla\cdot\boldsymbol{B})^2 - (\nabla\times\boldsymbol{E} + \dot{\boldsymbol{B}})^2\}\,.$$

From this we see that only $\boldsymbol{B}$ is dynamical. The Hamiltonian is

* This trouble tends to make the gauge fields themselves a nonlocal entity with a Dirac–Mandelstam string attached to it, see Eguchi [3]. The trick has also been proposed by Christ [7] in his theory of magnetic monopoles.

$$H = \tfrac{1}{2}(B^2 - E^2) + \frac{1}{2m^2}(\nabla \cdot B) + \frac{1}{2m^2}\dot{B}^2 - \frac{1}{2m^2}(\nabla \times E)^2$$

$$= \tfrac{1}{2}(B^2 - E^2) + \frac{1}{2m^2}(\nabla \cdot B)^2 + \frac{m^2}{2}\Pi^2 - \Pi \cdot (\nabla \times E)\,,$$

$$\Pi = \frac{1}{m}(\dot{B} + \nabla \times E)\,.$$

The equation of motion for $A_{\mu\nu}$ is

$$\left(1 - \frac{\Box}{m^2}\right)A_{\mu\nu} + \frac{1}{m^2}(\partial_\mu \partial_\lambda A_{\lambda\nu} - \partial_\nu \partial_\lambda A_{\lambda\mu}) = j_{\mu\nu}$$

from which follows

$$A_{\mu\nu} = \frac{1}{1 - \Box/m^2}\left\{ j_{\mu\nu} - \frac{\partial_\mu \partial_\lambda}{m^2} j_{\lambda\nu} - \frac{\partial_\nu \partial_\lambda}{m^2} j_{\lambda\mu} \right\}.$$

Eliminating $A_{\mu\nu}$ from L, we get an effective Lagrangian

$$L_{\text{eff}} = \tfrac{1}{2}\left\{ j_{\mu\nu} \frac{1}{1 - \Box/m^2} j_{\mu\nu} + \frac{2}{m^2}\partial_\mu j_{\mu\nu} \frac{1}{1 - \Box/m^2} \partial_\lambda j_{\lambda\nu} \right\}.$$

If the string is open, $\partial_\mu j_{\mu\nu}$ is nonvanishing at the end points of the string. This form is identical to the one in the Dirac–London case of magnetic confinement. It is also interesting that if we replace $j_{\mu\nu}$ in L with its dual, we get a similar result but with the wrong sign for the string energy.

References

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