

# Patenting or Secrecy: A Dynamic Analysis\*

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## Abstract

This paper studies firms' patenting decisions in a model where they sequentially discover a cost-reduction technology. A firm chooses between patenting and secrecy when her discovery occurs. Patenting grants an exclusive right with some probability while secrecy preserves lead time advantages over a stochastic period. In equilibrium, early innovators adopt secrecy and only a sufficiently late innovator patents. An innovator's incentive to patent is higher if the innovation arrival rate is higher. Two sources of inefficiency may exist in equilibrium: weak patent protection may delay patenting and strong patent protection may entitle an unnecessary monopoly. The socially efficient patent protection should be such that it just induces the first innovator to patent but not stronger. These findings may help explain why firms in Hi-tech industries find patenting attractive despite of weak industry patent protection, and suggest that the patent protection should be weaker in industries with higher innovation arrival rates or with more firms.

## 1 Introduction

An important strategic decision for a firm is how to protect its intellectual property. Firms often make heterogeneous choices of patenting or secrecy to

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protect their innovations. Recent surveys find that U.S. firms only patent a small proportion of their innovations. (See, e.g., Scherer, 1965; Pakes and Griliches, 1980; and Mansfield, 1986). Moreover, secrecy is viewed as an increasingly important way to appropriate innovations by firms (See, e.g., Levin et al., 1987 and Cohen et al., 2000). One natural question that arises is: why do some firms choose to patent while others adopt secrecy to protect and appropriate their innovations? An understanding of this question can help us evaluate whether the current patent system boosts innovations and shed light on the current debate of reforming U.S. patent system.

This paper attempts to answer this question. Our analysis is motivated by several observed features concerning innovations and patenting. First, multiple independent discoveries occur frequently. As discussed in Varian et al. (2005) and Shapiro (2007), this can happen because universal standards may restrict possible research paths, especially in network industries such as software, internet, telecommunications and payment media.

Second, patents are probabilistic. Many patent applications are not approved, and as emphasized in Lemley and Shapiro (2005), even issued patents could be ruled invalid through litigation.<sup>12</sup> Because of the requirement for full disclosure of innovation information during patent applications, once a patent application is denied or an issued patent is ruled invalid, rival firms can utilize the information to their benefit.

Third, if an innovator opts for secrecy it runs risk of allowing a later innovator to patent. Under the current U.S. patent laws, a later inventor is permitted to obtain a patent if a previous inventor abandoned, suppressed, or concealed its invention.<sup>3</sup> In addition, U.S. patent laws do not grant Prior User Right (PUR) which means the later inventor has absolute right to exclude the previous inventors who adopt secrecy. For instance, Garlock Inc. had discovered a process to create Tape of unsintered polytetrafluorethylene (PTFE) filament but decided to keep the innovation in secret use. However, the process was later rediscovered by W.L. Gore & Associates, Inc. (Gore) who obtained a patent for the process. Garlock Inc. tried to invalidate Gore's patent but the Court upheld the patent.<sup>4</sup> In another example, in early 1990, New England Biolabs and Bethesda Research Labs have pro-

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<sup>1</sup>Out of 484,955 patent applications received only 182,901 are granted patents in 2007. Source: U.S. Patent Statistics Chart Calendar Years 1963 - 2007. [http://www.uspto.gov/go/taf/us\\_stat.htm](http://www.uspto.gov/go/taf/us_stat.htm)

<sup>2</sup>Allison and Lemley (1998) report that, of the 300 final validity decisions in the date set, 138 (46%) find the patent invalid.

<sup>3</sup>See Merges and Duffy (2007).

<sup>4</sup>See Denicolo and Franzoni (2004) footnote 2 for detailed discussion.

duced the modified T7 DNA polymerase and offered it for sale but neither of the companies applied for a patent. However, the patent later was granted to Harvard researchers. They licensed the patent to U.S. Biochemicals which threaded New England Biolabs and Bethesda Research Labs of a lawsuit.<sup>5</sup>

To capture these features, we develop a dynamic model of innovation in which firms sequentially and independently discover a cost-reduction technology.<sup>6</sup> Discovery opportunities arrive according to a Poisson process. Firms that have made discoveries are referred as innovators. An innovator decides whether to patent or to keep the discovery as secret. If the innovator chooses the patenting option, it succeeds in being granted the patent only with some probability. Also, we assume that no PUR is granted, and thus by obtaining a patent, a latter innovator can exclude earlier innovators who opt for secrecy.

Taking into account the facts that multiple independent discoveries occur frequently, patents are probabilistic, and no PUR is granted an innovator's choice between patenting and secrecy becomes complicated. In particular, patenting, which initially seeks to exclude competitors, may actually help them by disclosing innovation information when the patent application is denied or the patent is ruled invalid. Adopting secrecy, which primarily wishes to hide the innovation for a long time, may end up with only a short lead if rivals make the same discovery quickly. As Cohen et al. (2000) report, information disclosure is one of the main reasons that innovation firms do not patent and blocking rival patents on related innovations prevails as a motive to patent.

We show that an innovator's decision on patenting crucially depends on whether its discovery occurs early or late. In equilibrium, early innovators opt for secrecy and only a sufficiently late innovator patents. A simple condition is provided to determine which innovator will patent. In the comparative statics, we show that an innovator's incentive to patent is higher if innovation arrival rate is higher. An increase number of firms encourage the early innovators to patent but reduce the patenting incentive of the later innovators. With more firms, patenting advances under a strong patent protection but delays when the patent protection is weak.

Our model also sheds light on the issues of socially efficient patent protection. Two sources of inefficiency may exist in equilibrium. First, a weak patent protection may delay patenting. Thus, the society has to endure a longer period of markets in which firms have strong market powers. Second,

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<sup>5</sup>See Eliot Marshall (1991).

<sup>6</sup>Our model can also be applied to product innovations.

a strong patent may entitle an unnecessary monopoly which reduces the social welfare. We show that the socially efficient patent protection should be such that it just induces the first innovator to patent but not stronger. Consequently, it suggests that the socially efficient patent protection should be weaker in industries with higher innovation rates or with more firms.

Finally, we extend our analysis to a model where the innovation arrival rate is endogenously determined by the R&D investment. We show how our equilibrium analysis carries over to the model with endogenous innovation arrival rate. In addition, we show that an increase in patent protection, under some condition, may impede the R&D investment and prevent innovation information disclosure.

Our paper is related to the literature of probabilistic patents. Departing from the traditional assumption of ironclad patents, Choi (1998) and Farrell and Shapiro (2008), among others, model patents as probabilistic.<sup>7</sup> Choi (1998) studies the incentive of a probabilistic patent holder to sue for infringement. Farrell and Shapiro (2008) investigate the private incentive for licensees to challenge a patent holder on the validity of the patent. These papers mainly examine the impacts of probabilistic patents in innovator-imitator or licensor-licensee framework. In contrast, we focus on a different aspect of the effect of probabilistic patents, namely, the effect on innovators' patenting decisions.

Our paper also relates to papers that study firms' decisions on patenting. Building on Horstmann et al. (1985), Anton and Yao (2004) construct a model in which an innovator with private information about the size of the innovation chooses between patent and secrecy as well as the degree of information disclosure. In their model, patents are used to signal the profitability of innovations. They show that innovations with small sizes are protected by patents with fully disclosed information while innovations with large sizes are opted for secrecy. In contrast, our model involves no signaling issue. Instead, we model multiple independent discoveries and study the strategic interactions among firms which are absent in their model.

Perhaps the paper closest to ours is Kultti et al. (2007). They develop an equilibrium search model of innovation and study innovators' choices on patenting versus secrecy as a part of the model. Like us, they model multiple independent discoveries. However, there are notable differences in terms of model settings and results. In their model, discoveries occur simultaneously and secrecy leaks out with an exogenous probability. The equilibrium concept in their one period model is Nash equilibrium. They

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<sup>7</sup>See Lemley and Shapiro (2005) for a survey on probabilistic patents.

show that patenting is a dominant strategy if patenting and secrecy provide a same level of protection. In our model, discoveries happen sequentially and secrecy fully protects an innovation until some other firms discover independently. The equilibrium concept is subgame perfect Nash equilibrium. We show that firms have heterogenous choices of patenting or secrecy.

The rest of the paper is organized as follows. Section 2 describes a model with exogenous innovation arrival rate. Equilibrium analysis is conducted in section 3. Section 4 performs the comparative statics. Socially efficient patent protection is discussed in section 5. Section 6 consider extensions to a model with endogenous arrival rate. Section 7 concludes. Proofs not in the text are collected in appendix.

## 2 The model

To illustrate the main idea of the paper, we start with the situation where the innovation arrival rate is exogenous which makes the analysis tractable and allows us to focus on firms' patenting decisions. We show that, in the section 6, how the results can be extended to the models where the innovation arrival rate is endogenously determined by the level of R&D investment which is chosen by firms.

Consider an industry with a fixed number,  $n$ , of ex-ante identical firms. The firms are about to discover a (drastic) cost-reduction technology.<sup>8</sup> The discovery process for each firm is assumed independent and identical, and is determined by a Poisson process with an exogenous innovation arrival rate  $\lambda$ .<sup>9</sup> Here, we consider the situations where firms randomly come up with "ideas" which could be turned into innovations with negligible costs.<sup>10</sup> It follows that firms will sequentially make discoveries according to a Poisson process with a collective industry innovation arrival rate.<sup>11</sup> Note that the industry innovation arrival rate varies overtime as firms make discoveries. In particular, let  $\Gamma_t(k)$  be the industry innovation arrival rate at time  $t$  when  $k$  firms race. Since each firm's discovery process is identical and independent it follows that  $\Gamma_t(k) = k\lambda$ . Hence, as more firms discover, the fewer firms remain to race and the industry innovation arrival rate is smaller which

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<sup>8</sup>The model can also be thought as diferrent but similar technologies which, however, are likely to be covered by one patent.

<sup>9</sup>Some researchers call it hit rate or hazard rate.

<sup>10</sup>See Scotchmer (2004).

<sup>11</sup>Note that the sum of independent Poisson processes is a Poisson process with the arrival rate equaling the sum of individual arrival rates. See Theorem 5.1 in Taylor and Karlin (1984).

implies the next industry discovery occurs later.

We shall refer *an innovator* as a firm that has made discovery. An innovator must decide whether to patent or adopt secrecy.<sup>12</sup> In contrast to the traditional literature where patents are ironclad, we assume that the patent in the model is probabilistic. With probability  $\alpha$ , the patent is with perfect protection. In such case, we call the patent is valid. With probability,  $1 - \alpha$ , we shall call the patent is invalid: either the patent application is denied or the patent is ruled invalid by court.<sup>13</sup> To isolate the driving forces behind the patenting decision and to make the analysis tractable, We make a technical assumption that the life of a valid patent is infinite.<sup>14</sup> Moreover, we normalize patent application fees to zero. However, the model can easily incorporate the case of a positive patenting fee,  $\tau$ , by scaling down the profit of patenting by  $\tau$ .

Firms earn profits from an output market. Since the cost reduction is drastic only firms that have made discoveries are able to compete. Assume that a firm is able to observe whether rival firms have discovered. Hence, all firms are able to correctly make decisions on production. A case in point is that a firm needs to buy machines to manufacture which is observable. In general, we do not rely on a specific structure of output market. Rather, we assume there is a reduced-form profit function that only depends on the number of firms that compete. Let  $\pi_i$  be the instantaneous profit for a single firm when  $i$  firms produce in output market. Throughout the paper, we assume that  $\pi_i$  is strictly decreasing and convex in  $i$ . The simplest example is Cournot competition with linear market demand and constant marginal production cost.<sup>15</sup> Three possible scenarios may appear, each of which determines the number of competing firms. (1) Under a valid patent, only the patentee produces and the profit is  $\pi_1$ ; (2) Under an invalid patent, all firms compete and profit for each firm is  $\pi_n$ ; (3)  $i$  firms have discovered and all

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<sup>12</sup>In our model, firms will not adopt secrecy and try to patent later. This is because the next innovation occurs stochastically and a firm faces a same decision problem. Moreover, the U.S. patent laws grant an inventor a grace period (one year) from an enabling disclosure date before an invention becomes ineligible for patent protection. See Merges and Duffy (2007). We assume that grace period is negligible.

<sup>13</sup>We assume that validity of a patent immediately reveals when a firm applies for patenting. See Farrell and Shapiro (2008) for detailed analysis of the incentive to challenge the validity of a patent.

<sup>14</sup>We will discuss the case of finite patent life in Appendix B.

<sup>15</sup>Assume market demand is  $P = a - bQ$  ( $a, b > 0$ ) where  $P$  and  $Q$  are market price and quantity, respectively. Let  $q$  the output for a single firm in the symmetric equilibrium. Assume marginal cost is  $c$ . Profit maximization and symmetric condition lead to  $q = \frac{a-c}{b(n+1)}$  and  $\pi_n = \frac{(a-c)^2}{b(n+1)^2}$ . One can verify that  $\frac{\partial \pi_n}{\partial n} < 0$  and  $\frac{\partial^2 \pi_n}{\partial n^2} > 0$ .

opt for secrecy. In this case,  $i$  firms compete and each earns  $\pi_i$ .

Since firms are ex-ante identical, without loss of generality, we index firms by their rank in discovery. Let innovator  $j$  (or firm  $j$ ) be the  $j$ th firm that makes discovery where  $j \in N$  and  $N = \{1, 2, \dots, n\}$ . Time is continuous. We shall call period  $j$  which begins when innovator  $j$  discovers and ends when innovator  $j + 1$  discovers. At the beginning of period  $j$ , innovator  $j$  discovers and decides whether to patent if no patent has been granted previously. If innovator  $j$  chooses to patent nature will determine the validity of the patent: with probability  $\alpha$ , the patent is valid; with probability  $1 - \alpha$ , the patent is invalid. Alternatively, innovator  $j$  may adopt secrecy. If secrecy is chosen, it moves on to period  $j + 1$  when innovator  $j + 1$  discovers and makes patenting decision.

The game in the model is a  $n$ -period dynamic game. The equilibrium concept is subgame perfect Nash equilibrium (SPNE). Given no previous patent has been granted, each innovator, taking into account the actions of successive innovators, chooses patenting or secrecy to maximize the expected profit. We show, in the next section, that, in equilibrium, an innovator's patenting decisions is a map from  $N$  into  $\{P, S\}$  where  $P$  and  $S$  stand for patenting and secrecy respectively.

### 3 Equilibrium Analysis

An innovator decides whether to patent and receives profit flows (profit streams) afterwards. Thus, a firm makes its patenting decision based on the discounted future profit from patenting and secrecy. Since innovator  $j$  will make patenting decision at the beginning of period  $j$  the future profit should be discounted to that point. Here, we derive some preliminary results which will be useful throughout the paper.

(i) Suppose that innovator  $j$  receives a flow of profit  $\pi$  through the entire period  $j$ . Let  $T_j$  denote the time length of period  $j$ . Note that  $T_j$  is distributed as a Poisson process with industry arrival rate  $\Gamma = (n - j)\lambda$ . Thus, it has probability density function,  $\Gamma e^{-\Gamma T_j}$ . Given a fixed time length  $T$ , the expected profit is:

$$\int_0^T \pi e^{-rt} dt = \frac{(1 - e^{-rT})}{r} \pi$$

Hence, for a random time length,  $T_j$ , the expected profit is for innovator

$j$  is:

$$\begin{aligned}
& \int_0^\infty \left( \int_0^{T_j} \pi e^{-rt} dt \right) (n-j) \lambda e^{-(n-j)\lambda T_j} dT_j \\
&= \int_0^\infty \frac{(1 - e^{-rT_j})}{r} \pi (n-j) \lambda e^{-(n-j)\lambda T_j} dT_j \\
&= \int_0^\infty \frac{(n-j)\lambda}{r} \pi \left[ e^{-(n-j)\lambda T_j} - e^{-rT_j - (n-j)\lambda T_j} \right] dT_j \\
&= \frac{(n-j)\lambda}{r} \pi \left[ \frac{1}{(n-j)\lambda} - \frac{1}{r + (n-j)\lambda} \right] \\
&= \frac{1}{r + (n-j)\lambda} \pi \\
&= \frac{1}{r} \theta_{n-j} \pi \tag{1}
\end{aligned}$$

where  $\theta_{n-j}$  is defined as

$$\theta_{n-j} = \frac{r}{r + (n-j)\lambda} \tag{2}$$

(ii) Suppose that innovator  $j$  receives an instantaneous profit  $\pi$  at the beginning of period  $j+1$ . Given a fixed time length,  $T$ , the expected profit is  $e^{-rT}\pi$ . Hence, for a random time length,  $T_j$ , the expected profit for innovator  $j$  is:

$$\begin{aligned}
\int_0^\infty e^{-rT_j} \pi (n-j) \lambda e^{-(n-j)\lambda T_j} dT_j &= \int_0^\infty \pi (n-j) \lambda e^{-rT_j - (n-j)\lambda T_j} dT_j \\
&= \frac{(n-j)\lambda}{r + (n-j)\lambda} \pi \\
&= (1 - \theta_{n-j}) \pi \tag{3}
\end{aligned}$$

(iii) Armed with the above results, from (1) and (3), we can show that if innovator  $j$  receives a flow of profit  $\pi$  in period  $h$  ( $h > j$ ) the expected profit is:

$$\frac{1}{r} \theta_{n-h} (1 - \theta_{n-h+1}) (1 - \theta_{n-h+2}) \cdots (1 - \theta_{n-j}) \pi \tag{4}$$

To see (4), note that the future profit discounted to the beginning of period  $h$  is  $\frac{1}{r} \theta_{n-h} \pi$ . Multiplying by  $(1 - \theta_{n-h+1})$  further discounts the profits to the beginning of period  $h-1$ . Applying the same logic, one can show (4) is the expected profit for innovator  $j$  at the beginning of period  $j$ .

### 3.1 Expected profit from patenting

Consider innovator  $j$ 's strategy given all previous innovators adopt secrecy. With probability  $\alpha$  innovator  $j$  obtains a valid patent and enjoys a monopoly profit. With probability  $1 - \alpha$  she has to share the profit with rivals under an invalid patent. Hence, the expected payoff for innovator  $j$  to patent is:

$$\begin{aligned}\Pi_p &= \int_0^\infty [\alpha\pi_1 + (1 - \alpha)\pi_n]e^{-rt} dt \\ &= \frac{1}{r} [\alpha\pi_1 + (1 - \alpha)\pi_n]\end{aligned}\tag{5}$$

Note that the expected profit from patenting is invariant with the rank in discovery,  $j$ . Moreover,  $\Pi_p$  does not depend on the strategies of successive innovators. The reason is that once a firm decides to patent the uncertainty on the validity of the patent reveals.

### 3.2 Expected profit from secrecy

If innovator  $j$  adopts secrecy the expected profit, unlike patenting, does depend on the strategies of successive innovators. Let  $\Pi_s(j|h)$  ( $h > j$ ) denote the expected payoff for innovator  $j$  opting for secrecy given innovator  $h$  patents. By (4),

$$\Pi_s(j|h) = \sum_{i=j}^{h-1} \frac{1}{r} \theta_{n-i} (1 - \theta_{n-i+1}) \cdots (1 - \theta_{n-j}) \pi_i + \frac{1}{r} (1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j}) (1 - \alpha) \pi_n\tag{6}$$

The first term (summation term) is the total expected profit of keeping secrecy from period  $j$  to period  $h - 1$ . The second term represents the expected profit from period  $h$  and on. When innovator  $h$  patents, with probability  $(1 - \alpha)$ , the patent is invalid and innovator  $j$  enjoys an infinite profit flow of  $\pi_n$ . By (3), the expected profit at the beginning of period  $j$  is  $(1 - \theta_{n-h}) \cdots (1 - \theta_{n-j}) \frac{1}{r} \pi_n$ .

Let

$$\Pi_s(j) = \Pi_s(j|j+1)$$

which denotes the expected profit if innovator  $j$  opts for secrecy given that innovator  $j + 1$  patents. By (6), we have

$$\Pi_s(j) = \frac{1}{r} \theta_{n-j} \pi_j + \frac{1}{r} (1 - \theta_{n-j}) (1 - \alpha) \pi_n\tag{7}$$

### 3.3 Equilibrium Choice of Patenting or Secrecy

To avoid the possibility of mixed strategies, we make the following assumption throughout the rest of the paper.

**Assumption 1** *A firm adopts secrecy if patenting and secrecy yield same expected profit.*

Alternatively, one could assume that a firm patents if receiving the same expected profit from patenting and secrecy. We next discuss an innovator's patenting strategy given the next innovation's action is patenting. In particular, given innovator  $j + 1$  patents, innovator  $j$  patents if and only if

$$\Pi_p > \Pi_s(j) \quad (8)$$

Define

$$\alpha_j = \frac{\pi_j - \pi_n}{\frac{\pi_1}{\theta_{n-j}} - \pi_n} \quad (9)$$

By (5) and (7), (8) becomes

$$\alpha > \alpha_j \quad (10)$$

Thus,  $\alpha_j$  is the cutoff value for innovator  $j$  to patent given innovator  $j + 1$  patents.

**Lemma 1**  *$\alpha_j$  strictly decreases in  $j$ .*

Lemma 1 states that the thresholds for innovators to patent given the next innovator patents decrease with the rank in discovery. Note that there are two factors that influence a firm's patenting decision. First, given the next innovator patents a firm receives a smaller size of profit from secrecy as more firms have discovered. Second, as fewer firms remain in racing to discover, the industry arrival rate decreases and the expected arrival time of next discovery delays. As a result, the firm adopting secrecy enjoys the profit flow longer. Lemma 1 shows that the first factor dominates the second. Consequently, a firm is more likely to patent if she discovers late knowing that the next innovator will patent.

Next, we turn to describing innovator  $j$ 's strategy when expecting innovator  $j + 1$  adopts secrecy.

**Lemma 2** *Given that innovator  $j + 1$ 's best strategy is to adopt secrecy innovator  $j$  opts for secrecy.*

Interestingly, given innovator  $j + 1$  adopts secrecy, the strategy of innovator  $j$  does not depend on the validity of the patent,  $\alpha$ . One immediate extension from Lemma 2 is that if an innovator optimally chooses secrecy in equilibrium all the previous innovators should also opt for secrecy. Combining Lemma 1 and 2, we are in a position to characterize the equilibrium of the model.

**Proposition 1** *Given  $\alpha$ , there exists a unique  $m$  such that  $\alpha_m < \alpha \leq \alpha_{m-1}$ . In equilibrium, innovator  $m$  patents while all previous innovators adopt secrecy.*

**Proof.** By Lemma 1,  $\{\alpha_j\}$  is a strictly decreasing sequence. Hence,  $[0, 1]$  can be divided by  $\{\alpha_j\}$  into non-overlapping intervals. Hence, given  $0 \leq \alpha \leq 1$ , a unique  $m$  can be found such that  $\alpha_m < \alpha \leq \alpha_{m-1}$ .

To show the second half of the proposition, we use backward induction. We first show that given no previous patent has been granted, innovator  $j$  with  $j \geq m$  chooses to patent. Consider the last innovator's choice. We have  $\Pi_s(n) = \frac{1}{r}\pi_n < \frac{1}{r}[\alpha\pi_1 + (1 - \alpha)\pi_n] < \Pi_p$ . Hence the last innovator will patent for sure because all rivals have discovered. Take one period back and consider the choice of innovator  $n - 1$ . Given innovator  $n$  patents and  $\alpha_{n-1} < \alpha$ , innovator  $n - 1$  chooses to patent. Applying the same logic repeatedly we can show that innovator  $j$  with  $j \geq m$  chooses to patent. Second, we show that given that innovator  $m$  patents, innovator  $j$  ( $j < m$ ) opts for secrecy. Since  $\alpha_m < \alpha \leq \alpha_{m-1}$  it follows that innovator  $m - 1$  chooses secrecy over patent. By Lemma 2, it is straightforward to show innovator  $j$  ( $j < m$ ) opts for secrecy. ■

Proposition 1 provides a simple characterization of the equilibrium. The intuition for the result that later innovators have more incentive to patent is as follows. As more rivals discover a firm makes less profit out of secrecy because of a more competitive output market. However, patenting provides invariant expected profit for all firms regardless of their ranks in discovery. As a consequence, early innovators enjoy more profit from secrecy and a sufficiently late innovator patents as profit from secrecy drops below that from the patent.

We illustrate the equilibrium by an example.

**Example 1** *Let  $n = 3$ ,  $\lambda = 0.1$ ,  $r = 0.2$ . Moreover, assume linear market demand  $P = a - bQ$  and constant marginal cost  $c$ . We have  $\alpha_1 = 0.43$ ,  $\alpha_2 = 0.16$ , and  $\alpha_3 = 0$ . If  $\alpha > \alpha_1$ , in equilibrium, innovator 1 patents. If  $\alpha_1 \geq \alpha > \alpha_2$ , in equilibrium innovator 1 adopts secrecy while innovator 2*

patents. If  $\alpha_2 \geq \alpha > \alpha_3$  in equilibrium, innovator 1 and innovator 2 adopt secrecy while innovator 3 patents.

**Proposition 2** *Expected profit decreases with the rank of discovery.*

In equilibrium, early innovators opt for secrecy to preserve lead time advantages. The profit earned diminishes when more firms discover. Therefore, early innovators enjoy a larger size of profit stream than the late innovators do. Once a firm chooses to patent, the lead time advantages disappear. Firms are placed at the same situation earning the same expected profit from the probabilistic patent.

## 4 Comparative Statics

In this section, we examine how changes in relevant parameters affect the equilibrium.

Proposition 1 shows that there is a unique  $m = m(\alpha, \lambda, n)$  such that innovator  $m$  patents and the previous innovators opt for secrecy. Define  $\rho(\alpha, \lambda, n)$  as the proportion of firms opting for secrecy:

$$\rho(\alpha, \lambda, n) = \frac{m(\alpha, \lambda, n) - 1}{n} \quad (11)$$

Since the industry arrival rate during period  $i$  is  $(n - i)\lambda$  the expected length of period  $i$  is

$$T_i = \frac{1}{(n - i)\lambda}$$

Define  $T(\alpha, \lambda, n)$  as the expected time when patenting occurs:

$$T(\alpha, \lambda, n) = E\left(\sum_{i=1}^{m-1} T_i\right) \quad (12)$$

We first see the impact of a change in patent protection  $\alpha$ .

**Proposition 3**  $m(\alpha, \lambda, n)$ ,  $\rho(\alpha, \lambda, n)$  and  $T(\alpha, \lambda, n)$  decrease in  $\alpha$ ;

Strengthening patent validity directly increases the profit from patenting. Moreover, it reduces the profit from secrecy because successive innovators have a higher chance of obtaining a valid patent. Therefore, a larger  $\alpha$  discourages firms from choosing secrecy and advances the timing of patenting.

We next see the impact of a change in innovation arrival rate  $\lambda$ .

**Proposition 4**  $m(\alpha, \lambda, n)$ ,  $\rho(\alpha, \lambda, n)$  and  $T(\alpha, \lambda, n)$  decrease in  $\lambda$ ;

A higher discovery arrival rate does not affect firms' profit from patenting. However, it shortens the length during which an innovator enjoys profit from secrecy because next discovery arrives more quickly. Thus, profit from secrecy decreases with  $\lambda$ . As a result, a larger  $\lambda$  induces early innovators to patent and patenting occurs earlier.

Proposition 4 has several implications. First, the result that firms prefer patenting with a larger  $\lambda$  may help explain why firms in Hi-tech industries find patenting attractive despite of relatively weak industry patent protection. This is because independent discoveries are likely to happen frequently in Hi-tech industries. Expecting that rivals would discover quickly a firm may find that adopting secrecy has little value and prefer patenting even under weak protection.

Second, as the innovation arrival rate becomes infinitely large, that is,  $\lambda \rightarrow \infty$ , the time lengths between discoveries become infinitely small. In other words, firms discover almost simultaneously. By (2),  $\theta_{n-j} \rightarrow 0$ . It follows, from (9), that  $\alpha_j \rightarrow 0$  for all  $j$ . This implies all firms choose to patent.

**Corollary 1** *When discoveries occur almost simultaneously all firms choose to patent.*

Kultti et al. (2007) obtain a similar result in a search equilibrium model of simultaneous innovations. However, the result in our model is achieved as a limiting result. In fact, the result can be obtained under a weaker condition. By (9)  $\alpha_1$  increases in  $\theta_1$  which, by (2), decreases in  $\lambda$  increases. Thus,  $\alpha_1$  decreases in  $\lambda$ . Moreover,  $\alpha_1 \rightarrow 0$  as  $\lambda \rightarrow \infty$ . Hence, by Lemma 1, all firms will choose to patent if the innovation arrival rate is sufficiently large.

**Corollary 2** *Given patent protection  $\alpha$  patenting is a dominant strategy if  $\lambda > \tilde{\lambda}$  where  $\tilde{\lambda} = \frac{1}{n-1} \frac{1-\alpha}{\alpha} r \left( \frac{\pi_1 - \pi_n}{\pi_1} \right)$ .*

Finally, we examine the impact of a change in the number of firms  $n$ .

**Lemma 3** *For each  $\alpha_j$  there exists a cutoff value,*

$$\lambda_j = \frac{r(\pi_1 - \pi_j)(\pi_n - \pi_{n+1})}{\pi_1[\pi_j - \pi_n - (n-j)(\pi_n - \pi_{n+1})]}$$

*such that  $\alpha_j$  increases in  $n$  when  $\lambda < \lambda_j$  but decreases in  $n$  when  $\lambda > \lambda_j$ . Moreover,  $\lambda_1 = 0$  and  $\lambda_j$  increases in  $j$ .*

Thus, with more firms, whether the incentive to patent for firm  $j$  increases or decreases depends on  $\lambda$ . Two effects tangle. First, an increase in the number of firms reduces the profit from patenting because competition is more severe under an invalid patent. We shall call it "free-ride effect" in the sense that firms take a free ride on the invalid patent. Second, profit from secrecy reduces with more firms because the expected arrival time of the next discovery shortens. We shall refer it as "racing effect" in the sense that firms are racing for discovery. The magnitudes of the two effects depend on the  $\lambda$  and  $j$ . Given  $j$  the racing effect is more prominent when  $\lambda$  is large. That leads to the first result in Lemma 3. Moreover, given  $\lambda$  the free ride effect is less significant for a larger  $j$  because the profit function is decreasing and convex in  $j$ . Thus, the cutoff value  $\lambda_j$  is higher for a larger  $j$ .

By the monotonicity of  $\lambda_j$ , it follows that for a given  $\lambda$ , as the number of firms increases, early innovators have higher incentive to patent while late innovators have lower incentive to patent. Thus, if patenting initially occurs early, which is associated with a strong patent protection, an increase in the number of firms will speed up the timing of patenting because racing effect dominates. On the other hand, if the patent protection is weak an increase in the number of firms causes a significant free ride effect which delays the patenting. We have the following proposition.

**Proposition 5** *There exists a  $\tilde{\alpha}$  such that, as the number of firms increases, patenting advances when  $\alpha > \tilde{\alpha}$  and delays when  $\alpha < \tilde{\alpha}$ .*

## 5 Socially Efficient Patent Protection

When considering choice between patenting and secrecy an innovator does not internalize the effect on the consumer surplus and the other firms' profits. Thus, private returns may differ from the social returns. This section addresses the following question: given the number of firms in an industry ( $n$ ) and innovation arrival rate ( $\lambda$ ), what is the patent protection ( $\alpha$ ) that leads to socially efficient outcomes?

Let  $S_k$  be the instantaneous social welfare when  $k$  firms produce where instantaneous social welfare is defined as the sum of consumer surplus and producer surplus. We make the following assumption.

**Assumption 2** *The instantaneous social welfare strictly increases with the number of firms, that is,  $S_{k_1} > S_{k_2}$  if  $k_1 > k_2$ .*

Suppose, given  $\alpha$ , innovator  $m$  patents in the equilibrium. Define total social welfare,  $TS(\alpha)$ , as the weighted sum of instantaneous social welfare:

$$\begin{aligned}
TS(\alpha) &= \frac{1}{r}\theta_{n-1}S_1 \\
&\quad + \frac{1}{r}\theta_{n-2}(1 - \theta_{n-1})S_2 \\
&\quad + \frac{1}{r}\theta_{n-3}(1 - \theta_{n-2})(1 - \theta_{n-1})S_3 \\
&\quad + \dots \\
&\quad + \frac{1}{r}\theta_{n-m+1}(1 - \theta_{n-m+2}) \dots (1 - \theta_{n-1})S_{m-1} \\
&\quad + \frac{1}{r}(1 - \theta_{n-m+1})(1 - \theta_{n-m+2}) \dots (1 - \theta_{n-1})[\alpha S_1 + (1 - \alpha)S_n]
\end{aligned} \tag{13}$$

The first  $m - 1$  lines are the expected social welfare in the first  $m - 1$  periods when innovators opt for secrecy while the last line is the expected social welfare when innovator  $m$  patents.

A social planner's objective function is

$$\max_{\alpha \in [0,1]} TS(\alpha) \tag{14}$$

Consider an increase in patent protection from  $\alpha_1$  to  $\alpha_2$  such that  $\alpha_{m-1} < \alpha_1 < \alpha_2 \leq \alpha_m$ . In words, the change in  $\alpha$  is small and does not alter  $m$  in equilibrium. The effect of the increase in  $\alpha$  is nothing but a higher chance of a monopoly for innovator  $m$ . As a consequence, the total social welfare decreases.

**Lemma 4** *Total social welfare can be increased if a reduction of  $\alpha$  results in the same  $m$  in equilibrium.*

We can greatly cut down the set of possible  $\alpha$  leading to total social welfare maximization. According to Lemma 3, total social welfare maximization has to occur when  $\alpha$  just induces a switch in  $m$ . In other words, it happens at one of the  $\{\alpha_j\}$  ( $j \in N$ ). Hence, we only need to compare the  $n$  possible equilibrium outcomes. The maximization problem is simplified to

$$\max_{\alpha \in \Omega(\alpha)} TS(\alpha) \quad \text{where} \quad \Omega(\alpha) = \{\alpha_j, j \in N\} \tag{15}$$

The social planner chooses a  $\alpha_j$  to maximize the total social welfare. However, each  $\alpha_j$  is uniquely associated with a  $m$ . Therefore, it is as if the social planner chooses  $m$  to maximize the total social welfare. The next proposition states that the socially efficient patent protection should induce the first innovator to patent.

**Proposition 6** *The socially efficient patent policy,  $\alpha^*$ , that maximizes the total social welfare should be such that it just induces the first innovator to patent. That is,  $\alpha^* = \alpha_1$ .*

Given a level of patent protection. Two sources of inefficiency may exist. First, a weak patent, by Proposition 3, delays timing of patenting. Thus, the society has to endure a longer period of markets in which firms have strong market powers. Second, a strong patent may give an unnecessary monopoly power which reduces the social welfare.

From (2), we have  $\theta_1 = \frac{\tau}{\tau+\lambda} < 1$ . Hence, from (9),  $\alpha_1 < 1$ . We have the following corollary.

**Corollary 3** *Full patent protection is never socially efficient for  $n > 1$ .*

When a monopoly exists in the innovation market it asks for full protection in exchange for revealing the innovation information because it does not face any potential threat. Situation changes in an oligopoly market. If the first innovator opts for secrecy it could be potentially excluded by a later innovator who obtains a patent. Thus, the profit from secrecy decreases. It follows that the compensation to induce patenting is less than a full protection.

Since  $\alpha_1$  depends on  $\lambda$  and  $n$  we have the following proposition.

**Proposition 7**  *$\alpha^*$  decreases, respectively, in  $\lambda$  and  $n$ .*

The proposition has implications on patent policies: the socially efficient patent protection should vary with the rate of innovation and the number of firms in an industry. In particular, the socially efficient patent protection should be weaker for industries with higher innovation arrival rate or with more firms.

## 6 A model with endogenous arrival rate

We have focused on the situation where the innovation arrival rate is exogenously determined. In this section, we extend the analysis to a model where

the innovation arrival rate is endogenously determined. In particular, we adopt the framework as in Loury (1979) considering firms pay an up-front R&D investment which generates a steady flow of innovation arrival rate.

The game has two stages. At stage 1, each firm chooses R&D investment level  $c(\lambda^i)$  which yields an innovation arrival rate  $\lambda^i$ . At stage 2, firms make discoveries sequentially and each innovator decides whether to patent. We only consider the equilibrium where firms adopt symmetric strategy at stage 1. That is, each firm chooses a same level of R&D investment which we denote as  $c(\lambda^*)$ . Hence, the stage 2 is identical to the model in section 2 except that the innovation arrival rate is  $\lambda^*$  which is chosen by firms at stage 1. Therefore, the equilibrium analysis in section 3 carries over to the model with endogenous arrival rate. We next explore firm's R&D decision at the stage 1.

Recall that stage 2 consists  $n$  periods and period  $j$  begins when innovator  $j$  discovers and ends when innovator  $j + 1$  discovers. Let  $V_W^i$  ( $V_L^i$ ) be the discounted value of future profits from succeeding (not succeeding) in  $i$ th period. Suppose in equilibrium innovator  $m$  chooses to patent and the previous innovators adopt secrecy. Given other firms choose  $\lambda^*$  a firm choose  $\lambda$  to maximize:

$$\begin{aligned} V(\lambda) &= \int_0^\infty e^{-rt} e^{-[\lambda+(n-1)\lambda^*]t} [\lambda V_W + (n-1)\lambda^* V_L] dt - c(\lambda) \\ &= \frac{\lambda V_W^1 + (n-1)\lambda^* V_L^1(\lambda)}{r + \lambda + (n-1)\lambda^*} - c(\lambda) \end{aligned} \quad (16)$$

where

$$\begin{aligned} V_L^i &= \frac{\lambda V_W^{i+1} + (n-i-1)\lambda^* V_L^{i+1}}{r + \lambda + (n-i-1)\lambda^*} & i = 1, \dots, m-1 \\ V_L^m &= \frac{1}{r} (1 - \alpha) \pi_n \\ V_W^i &= \Pi_s(j|m) & i = 1, \dots, m-1 \\ V_W^m &= \Pi_p \end{aligned}$$

To see the above expressions, note that, in any period  $i$ , given no firm has succeeded yet and rival firms choose  $\lambda^*$ , with an instantaneous probability  $\frac{\lambda}{r+\lambda+(n-1)\lambda^*}$ , a firm discovers and receive  $V_W^i$  and with an instantaneous probability  $\frac{(n-i-1)\lambda^*}{r+\lambda+(n-i-1)\lambda^*}$  one of the rivals makes a discovery the firm's payoff is  $V_L^i(\lambda)$ . Since the first  $m-1$  innovators adopt secrecy  $V_L^i(\lambda)$  is the value

of entering into the next period where  $(n - i - 1)$  rivals exist. Moreover, innovator  $m$  patents and receives  $\Pi_p$  defined in (5). The previous innovator  $j$  adopt secrecy and the payoff is  $\Pi_s(j|m)$  which is determined by (6)

Note that  $V_W^i$  is independent of  $\lambda$ . First order condition, letting  $\lambda = \lambda^*$ , yield,

$$\frac{rV_W^1 + (n-1)\lambda^* [V_W^1 - V_L^1(\lambda^*)] + (n-1)\lambda^* (r+n\lambda^*) \frac{\partial V_L^1(\lambda^*)}{\partial \lambda}}{(r+n\lambda^*)^2} - c'(\lambda^*) = 0 \quad (17)$$

and

$$\begin{aligned} & \frac{\partial V_L^i(\lambda)}{\partial \lambda} \\ = & \frac{rV_W^{i+1} + (n-i-1)\lambda^* [V_W^{i+1} - V_L^{i+1}(\lambda^*)] + (n-i-1)\lambda^* (r+(n-i)\lambda^*) \frac{\partial V_L^{i+1}(\lambda^*)}{\partial \lambda}}{[r+(n-i)\lambda^*]^2} \end{aligned} \quad (18)$$

The equilibrium  $\lambda^*(\alpha)$  is implicitly defined by (17) and (18).

**Proposition 8** *In the model with endogenous arrival rate, given  $\alpha$ , there exists a unique  $m$  such that  $\alpha_m < \alpha \leq \alpha_{m-1}$ . In equilibrium, innovator  $m$  patents while previous innovators adopt secrecy. Moreover, firms choose  $\lambda^*$  which is determined by (17) and (18).*

We next investigate how patent protection affect the innovation incentive. To illustrate, we consider an example where  $n = 2$  and output market competition is Bertrand fashion which implies  $\pi_2 = 0$ . Moreover, assume that  $r = 0.2$ ,  $\pi_1 = 1$  and  $c(\lambda) = \lambda^2$ . We find that  $\alpha_1 = 0.31$ . Thus, in equilibrium firm 1 patents if  $\alpha > \alpha_1$ . In stage 1, the firm's problem is

$$\max \frac{\lambda \left(\frac{1}{r}\alpha\right)}{r + \lambda + \lambda^*} - \lambda^2$$

First order condition and symmetry yield

$$\frac{(r + \lambda^*) \frac{1}{r}\alpha}{(r + 2\lambda^*)^2} - 2\lambda^* = 0$$

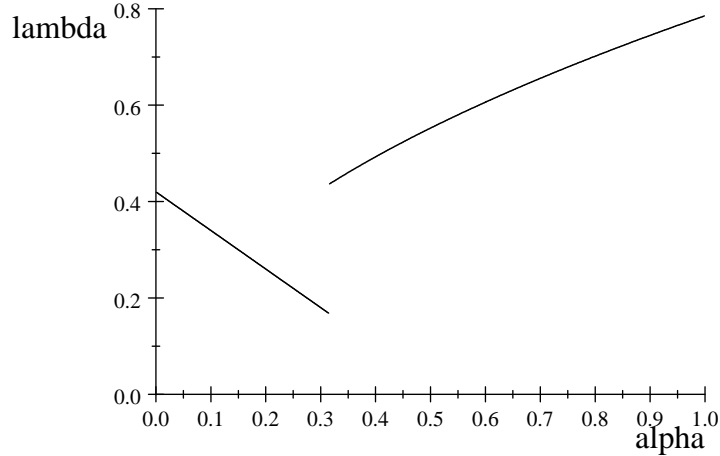
If  $\alpha \leq \alpha_1$  in equilibrium, firm 1 adopt secrecy and firm 2 patents, the firm's problem is

$$\max \frac{\lambda \left(\frac{1}{r+\lambda^*}\right) + \lambda^* \left(\frac{1}{r}\alpha \frac{\lambda}{r+\lambda}\right)}{r + \lambda + \lambda^*} - \lambda^2$$

First order condition is

$$\frac{1}{r(r + \lambda^*)^2 (r + 2\lambda^*)^2} (r\lambda^{*2} + 2r^2\lambda^* - \alpha\lambda^{*3} + r^3 + r\alpha\lambda^{*2} + r^2\alpha\lambda^*) - 2\lambda^* = 0$$

The following graph shows how  $\lambda^*$  changes with  $\alpha$ .



Interestingly, one can see that when the patent protection is initially weak ( $\alpha < 0.31$ ) an increase in  $\alpha$  actually leads to a lower  $\lambda^*$ . Hence, strengthening the patent protection may not necessarily encourage R&D investment.

**Claim 1** *In the model with endogenous arrival rate, as patent protection becomes stronger, the innovation incentive can decrease.*

The intuition is as follows. When the patent protection is weak the equilibrium is that the first innovator adopts secrecy and second innovator chooses to patent. An increase in the patent protection, as long as it does not induce the first innovator to patent, will not affect  $V_W^1$  but increase  $V_L^1$ . In other words, the payoff difference of being the first innovator and the second innovator becomes smaller. This reduces the incentive of R&D investment.<sup>16</sup>

In the above example, when  $\alpha < \alpha_1$  a stronger patent protection decreases  $\lambda^*$  which will delay the occurrence of the innovations. As a corollary, we have

**Corollary 4** *In the model with endogenous arrival rate, a stronger patent can delay patenting.*

<sup>16</sup>The result that an increase in patent protection may reduce R&D investment has also been found in the literature. For example, Ganilli (1992) and Choi (1998).

## 7 Conclusion

The heterogeneous choices of patenting and secrecy are well documented in empirical literature of innovation. We have shown that it can arise as market equilibrium when discoveries occur sequentially. In particular, given the number of firms, innovation arrival rate and patent protection, each firm's decision is uniquely determined by its rank in discovery. Innovators have more incentive to patent in the environment with higher innovation arrival rate. The result may help explain the puzzle that firms in Hi-tech industries actively seek for patenting even the patent protection is weak.

Total social welfare is maximized when the patent protection is set such that it just induces the first innovator to patent but not stronger. Thus, a strong patent is not necessarily beneficial from society's point of view. Indeed, socially efficient patent protection should be weaker for industries with higher innovation arrival rates or with more firms.

Our model assumes no PUR as in the current U.S. patent system. Instead, if assuming that the patent laws do grant PUR as in European patent system we expect to see quite different results. Without PUR a valid patent holder is entitled to exclude all other firms including early innovators opting for secrecy. One implication is that patenting gives the same expected profit which is independent of the rank in discovery. However, with PUR, profit from patenting is no longer independent on the rank of discovery. In fact, it decreases with the rank of discovery because PUR allows early innovators to share profit with a late patenting innovator. Under the PUR system, an innovator may still use a patent as a device to exclude the successive innovators. The effect is more prominent at the early stage when a large number of firms still race for discoveries. In other words, late innovators gain less from patenting than early innovators do. Thus, the equilibrium may differ from the one without PUR. As one direction of future research one would be interested to compare the equilibrium, profit and welfare with and without PUR.

We conclude by pointing out some possible directions for future research. One possibility of future research is to allow firms to be asymmetric. For example, one could adjust to the models with a dominant firm in innovation ability or an incumbent in output market to see how the asymmetry of firms affects patenting decision. Second, as discussed above, it would be interesting to compare patent systems with and without PUR to see how they affect firms' patenting decisions and which system is better from the social point of view.

## A Appendix

### A.1 Proof of Lemma 1

**Proof.** Since  $\pi_i$  is strictly decreasing and convex in  $j$  we have

$$\left(\frac{n-j-1}{n-j}\right)\pi_j + \left(\frac{1}{n-j}\right)\pi_n > \pi_{j+1}$$

thus,

$$(n-j-1)\pi_j - (n-j)\pi_{j+1} + \pi_n > 0$$

By (9)

$$\begin{aligned} & \alpha_j - \alpha_{j+1} \\ = & \frac{\pi_j - \pi_n}{\frac{\pi_1}{\theta_{n-j}} - \pi_n} - \frac{\pi_{j+1} - \pi_n}{\frac{\pi_1}{\theta_{n-j-1}} - \pi_n} \\ = & \frac{(\pi_j - \pi_n) \left(\frac{\pi_1}{\theta_{n-j-1}} - \pi_n\right) - (\pi_{j+1} - \pi_n) \left(\frac{\pi_1}{\theta_{n-j}} - \pi_n\right)}{\left(\frac{\pi_1}{\theta_{n-j}} - \pi_n\right) \left(\frac{\pi_1}{\theta_{n-j-1}} - \pi_n\right)} \\ = & \frac{(\pi_j - \pi_n) \left(\frac{r+(n-j-1)\lambda}{r}\pi_1 - \pi_n\right) - (\pi_{j+1} - \pi_n) \left(\frac{r+(n-j)\lambda}{r}\pi_1 - \pi_n\right)}{\left(\frac{\pi_1}{\theta_{n-j}} - \pi_n\right) \left(\frac{\pi_1}{\theta_{n-j-1}} - \pi_n\right)} \\ = & \frac{(\pi_j - \pi_n) \left(\frac{(n-j-1)\lambda}{r}\pi_1 + \pi_1 - \pi_n\right) - (\pi_{j+1} - \pi_n) \left(\frac{(n-j)\lambda}{r}\pi_1 + \pi_1 - \pi_n\right)}{\left(\frac{\pi_1}{\theta_{n-j}} - \pi_n\right) \left(\frac{\pi_1}{\theta_{n-j-1}} - \pi_n\right)} \\ = & \frac{\frac{(n-j-1)\lambda}{r}\pi_1(\pi_j - \pi_n) + (\pi_j - \pi_n)(\pi_1 - \pi_n) - \frac{(n-j)\lambda}{r}\pi_1(\pi_{j+1} - \pi_n) - (\pi_{j+1} - \pi_n)(\pi_1 - \pi_n)}{\left(\frac{\pi_1}{\theta_{n-j}} - \pi_n\right) \left(\frac{\pi_1}{\theta_{n-j-1}} - \pi_n\right)} \\ = & \frac{\frac{\lambda}{r}\pi_1 [(n-j-1)(\pi_j - \pi_n) - (n-j)(\pi_{j+1} - \pi_n)] + (\pi_j - \pi_{j+1})(\pi_1 - \pi_n)}{\left(\frac{\pi_1}{\theta_{n-j}} - \pi_n\right) \left(\frac{\pi_1}{\theta_{n-j-1}} - \pi_n\right)} \\ = & \frac{\frac{\lambda}{r}\pi_1 [(n-j-1)\pi_j - (n-j)\pi_{j+1} + \pi_n] + (\pi_j - \pi_{j+1})(\pi_1 - \pi_n)}{\left(\frac{\pi_1}{\theta_{n-j}} - \pi_n\right) \left(\frac{\pi_1}{\theta_{n-j-1}} - \pi_n\right)} > 0 \end{aligned}$$

■

### A.2 Proof of Lemma 2

**Proof.** We first briefly outline the proof as follows. First, we show that firm  $j$  earns more profit than firm  $j+1$  does when they both adopt secrecy

regardless of the actions of the successive innovators. Second, patenting gives both firms the same expected profit as described in (5). Therefore, it is optimal for firm  $j$  to choose secrecy over patent if it is optimal for firm  $j + 1$  to do so.

We turn the details of the proof. Since the game in the model is with complete information all firms correctly expect the strategies of their successive firms. Suppose it is expected that firm  $h > j + 1$  will patent when she discovers. The expected profit from secrecy for firm  $j$  and firm  $j + 1$  are, respectively:

$$\Pi_s(j|h) = \sum_{i=j}^{h-1} \frac{1}{r} \theta_{n-i} (1 - \theta_{n-i+1}) \cdots (1 - \theta_{n-j}) \pi_i + \frac{1}{r} (1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j}) (1 - \alpha) \pi_n$$

and

$$\Pi_s(j+1|h) = \sum_{i=j+1}^{h-1} \frac{1}{r} \theta_{n-i} (1 - \theta_{n-i+1}) \cdots (1 - \theta_{n-j-1}) \pi_i + \frac{1}{r} (1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j-1}) (1 - \alpha) \pi_n$$

Let

$$\beta_i = \frac{1}{r} \theta_{n-i} (1 - \theta_{n-i+1}) \cdots (1 - \theta_{n-j})$$

and

$$\delta_i = \frac{1}{r} \theta_{n-i} (1 - \theta_{n-i+1}) \cdots (1 - \theta_{n-j-1})$$

Note that

$$\begin{aligned} & \sum_{i=j}^{h-1} \beta_i + \frac{1}{r} (1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j}) \\ &= \frac{1}{r} \theta_{n-j} + \frac{1}{r} \theta_{n-j-1} (1 - \theta_{n-i}) + \\ & \quad + \cdots + \frac{1}{r} \theta_{n-h+1} (1 - \theta_{n-h+2}) \cdots (1 - \theta_{n-j}) + \frac{1}{r} (1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j}) \\ &= \frac{1}{r} \end{aligned}$$

and

$$\begin{aligned}
& \sum_{i=j+1}^{h-1} \delta_i + \frac{1}{r}(1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j-1}) \\
&= \frac{1}{r}\theta_{n-j-1} + \frac{1}{r}\theta_{n-j-2}(1 - \theta_{n-i-1}) + \\
& \quad + \cdots + \frac{1}{r}\theta_{n-h+1}(1 - \theta_{n-h+2}) \cdots (1 - \theta_{n-j-1}) + \frac{1}{r}(1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j-1}) \\
&= \frac{1}{r}
\end{aligned}$$

We have

$$\Pi_s(j|h) = \sum_{i=j}^{h-1} \beta_i \pi_i + \left(\frac{1}{r} - \sum_{i=j}^{h-1} \beta_i\right)(1 - \alpha)\pi_n$$

and

$$\Pi_s(j+1|h) = \sum_{i=j+1}^{h-1} \delta_i \pi_i + \left(\frac{1}{r} - \sum_{i=j+1}^{h-1} \delta_i\right)(1 - \alpha)\pi_n$$

Moreover,

$$\begin{aligned}
\beta_j + \beta_{j+1} - \delta_{j+1} &= \frac{1}{r}\theta_{n-j} + \frac{1}{r}\theta_{n-j-1}(1 - \theta_{n-j}) - \frac{1}{r}\theta_{n-j-1} \\
&= \frac{1}{r}\theta_{n-j}(1 - \theta_{n-j-1}) > 0
\end{aligned}$$

and

$$\begin{aligned}
\sum_{i=j}^{h-1} \beta_i &= \frac{1}{r} - \frac{1}{r}(1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j-1})(1 - \theta_{n-j}) \\
&> \frac{1}{r} - \frac{1}{r}(1 - \theta_{n-h+1}) \cdots (1 - \theta_{n-j-1}) \\
&= \sum_{i=j+1}^{h-1} \delta_i
\end{aligned}$$

Hence,

$$\begin{aligned}
& \Pi_s(j|h) - \Pi_s(j+1|h) \\
&= \beta_j \pi_j + \sum_{i=j+1}^{h-1} (\beta_i - \delta_i) \pi_i - \left( \sum_{i=j}^{h-1} \beta_i - \sum_{i=j+1}^{h-1} \delta_i \right) (1 - \alpha) \pi_n \\
&> (\beta_j + \beta_{j+1} - \delta_{j+1}) \pi_{j+1} + \sum_{i=j+2}^{h-1} (\beta_i - \delta_i) \pi_i - \left( \sum_{i=j}^{h-1} \beta_i - \sum_{i=j+1}^{h-1} \delta_i \right) (1 - \alpha) \pi_n \\
&> \left( \sum_{i=j}^{h-1} \beta_i - \sum_{i=j+1}^{h-1} \delta_i \right) \pi_{h-1} + \left( \sum_{i=j}^{h-1} \beta_i - \sum_{i=j+1}^{h-1} \delta_i \right) (1 - \alpha) \pi_n \\
&= \left( \sum_{i=j}^{h-1} \beta_i - \sum_{i=j+1}^{h-1} \delta_i \right) [\pi_{h-1} - (1 - \alpha) \pi_n] > 0
\end{aligned}$$

Thus

$$\Pi_s(j|h) > \Pi_s(j+1|h)$$

Given firm  $j+1$  optimally opts for secrecy, we have

$$\Pi_s(j+1|h) > \Pi_p$$

It follows that

$$\Pi_s(j|h) > \Pi_p$$

That is firm  $j$  opts for secrecy. ■

### A.3 Proof of Proposition 2

**Proof.** Suppose firm  $m$  patents. By Proposition 1, firm  $i < m$  opts for secrecy. By Lemma 2,

$$\Pi_s(j|m) > \Pi_s(j+1|m)$$

Hence, the expected profit decreases with  $j$  when  $j < m$ . In addition,  $\Pi_p$  is the expected profit for firm  $m$ . Firm  $m-1$  opts for secrecy which implies

$$\Pi_s(m-1) > \Pi_p$$

Finally, firm  $j$  with  $j > m$  earns

$$\begin{aligned}
\frac{1}{r} (1 - \alpha) \pi_n &< \frac{1}{r} [\alpha \pi_1 + (1 - \alpha) \pi_n] \\
&= \Pi_p
\end{aligned}$$

■

#### A.4 Proof of Proposition 3

**Proof.** Suppose  $\hat{\alpha} > \alpha$ . Let  $m = m(\alpha, \lambda, n)$  and  $\hat{m} = m(\hat{\alpha}, \lambda, n)$ . In equilibrium, we have  $\alpha_m < \alpha \leq \alpha_{m-1}$ . Hence,  $\alpha_m \leq \hat{\alpha}$ . Since a change in  $\alpha$  does not change  $\alpha_j$  and  $\hat{\alpha} \in (\alpha_{\hat{m}}, \alpha_{\hat{m}-1}]$  it follows that  $\alpha_m \leq \alpha_{\hat{m}}$ . By (ii) in lemma 3,  $\hat{m} \leq m$ . Hence,  $\hat{\rho} \leq \rho$ . Let  $T = T(\alpha, \lambda, n)$  and  $\hat{T} = T(\hat{\alpha}, \lambda, n)$ . When  $\hat{m} \leq m$ , we have  $\hat{T} \leq T$ . ■

#### A.5 Proof of Proposition 4

**Proof.** We first show that  $\alpha_j$  decreases with  $\lambda$ . From equation (9), one can show that  $\alpha_j$  increases in  $\theta_{n-j}$ . In addition, by (2),  $\theta_{n-j}$  decreases in  $\lambda$ . Therefore,  $\alpha_j$  decreases in  $\lambda$ .

Hence, given  $\hat{\lambda} > \lambda$ , we have  $\alpha_j(\hat{\lambda}) < \alpha_j(\lambda)$ . Given  $\lambda$ , in equilibrium,  $\alpha_m(\lambda) < \alpha \leq \alpha_{m-1}(\lambda)$ . Hence,  $\alpha > \alpha_m(\hat{\lambda})$ . Let  $\hat{m} = m(\alpha, \hat{\lambda}, n)$  and  $\hat{\rho} = \rho(\alpha, \hat{\lambda}, n)$ . Given  $\hat{\lambda}$ , we have  $\hat{m} \leq m$ . Hence,  $\hat{\rho} \leq \rho$ . Let  $\hat{T} = T(\alpha, \hat{\lambda}, n)$ . When  $\hat{m} \leq m$  and  $\hat{\lambda} > \lambda$ , we have  $\hat{T} \leq T$ . ■

#### A.6 Proof of lemma 3

**Proof.**

$$\begin{aligned}
& \alpha_j(n) - \alpha_j(n+1) \\
= & \frac{\pi_j - \pi_n}{\frac{\pi_1}{\theta_{n-j}} - \pi_n} - \frac{\pi_j - \pi_{n+1}}{\frac{\pi_1}{\theta_{n+1-j}} - \pi_{n+1}} \\
= & \frac{\pi_j - \pi_n}{\frac{(n-j)\lambda}{r}\pi_1 + \pi_1 - \pi_n} - \frac{\pi_j - \pi_{n+1}}{\frac{(n+1-j)\lambda}{r}\pi_1 + \pi_1 - \pi_{n+1}} \\
= & \frac{(\pi_j - \pi_n) \left[ \frac{(n+1-j)\lambda}{r}\pi_1 + \pi_1 - \pi_{n+1} \right] - (\pi_j - \pi_{n+1}) \left[ \frac{(n-j)\lambda}{r}\pi_1 + \pi_1 - \pi_n \right]}{\left[ \frac{(n-j)\lambda}{r}\pi_1 + \pi_1 - \pi_n \right] \left[ \frac{(n+1-j)\lambda}{r}\pi_1 + \pi_1 - \pi_{n+1} \right]} \\
= & \frac{(\pi_j - \pi_n) \frac{(n+1-j)\lambda}{r}\pi_1 + (\pi_j - \pi_n)(\pi_1 - \pi_{n+1}) - (\pi_j - \pi_{n+1}) \frac{(n-j)\lambda}{r}\pi_1 - (\pi_j - \pi_{n+1})(\pi_1 - \pi_n)}{\left[ \frac{(n-j)\lambda}{r}\pi_1 + \pi_1 - \pi_n \right] \left[ \frac{(n+1-j)\lambda}{r}\pi_1 + \pi_1 - \pi_{n+1} \right]} \\
= & \frac{[(n+1-j)(\pi_j - \pi_n) - (n-j)(\pi_j - \pi_{n+1})] \frac{\pi_1}{r}\lambda + (\pi_j - \pi_n)(\pi_1 - \pi_{n+1}) - (\pi_j - \pi_{n+1})(\pi_1 - \pi_n)}{\left[ \frac{(n-j)\lambda}{r}\pi_1 + \pi_1 - \pi_n \right] \left[ \frac{(n+1-j)\lambda}{r}\pi_1 + \pi_1 - \pi_{n+1} \right]} \\
= & \frac{[(\pi_j - \pi_n) - (n-j)(\pi_n - \pi_{n+1})] \frac{\pi_1}{r}\lambda - (\pi_1 - \pi_j)(\pi_n - \pi_{n+1})}{\left[ \frac{(n-j)\lambda}{r}\pi_1 + \pi_1 - \pi_n \right] \left[ \frac{(n+1-j)\lambda}{r}\pi_1 + \pi_1 - \pi_{n+1} \right]}
\end{aligned}$$

When  $\lambda > \lambda_j = \frac{r(\pi_1 - \pi_j)(\pi_n - \pi_{n+1})}{\pi_1[\pi_j - \pi_n - (n-j)(\pi_n - \pi_{n+1})]}$  we have

$$\alpha_j(n) > \alpha_j(n+1)$$

and when  $\lambda < \lambda_j$  we have

$$\alpha_j(n) < \alpha_j(n+1)$$

It is straightforward to show that  $\lambda_1 = 0$ . To show  $\{\lambda_j\}$  increases in  $j$ , note that

$$\begin{aligned} & \lambda_j - \lambda_{j+1} \\ = & \frac{r(\pi_1 - \pi_j)(\pi_n - \pi_{n+1})}{\pi_1[\pi_j - \pi_n - (n-j)(\pi_n - \pi_{n+1})]} - \frac{r(\pi_1 - \pi_{j+1})(\pi_n - \pi_{n+1})}{\pi_1[\pi_{j+1} - \pi_n - (n-j-1)(\pi_n - \pi_{n+1})]} \\ = & \frac{r(\pi_n - \pi_{n+1})}{\pi_1} \left[ \frac{\pi_1 - \pi_j}{\pi_j - \pi_n - (n-j)(\pi_n - \pi_{n+1})} - \frac{\pi_1 - \pi_{j+1}}{\pi_{j+1} - \pi_n - (n-j-1)(\pi_n - \pi_{n+1})} \right] \\ = & \frac{r(\pi_n - \pi_{n+1})}{\pi_1[\pi_j - \pi_n - (n-j)(\pi_n - \pi_{n+1})][\pi_{j+1} - \pi_n - (n-j-1)(\pi_n - \pi_{n+1})]} \times \\ & [(\pi_1 - \pi_j)[\pi_{j+1} - \pi_n - (n-j-1)(\pi_n - \pi_{n+1})] - (\pi_1 - \pi_{j+1})[\pi_j - \pi_n - (n-j)(\pi_n - \pi_{n+1})]] \end{aligned}$$

Thus,

$$\begin{aligned} & \text{sign}(\lambda_j - \lambda_{j+1}) \\ = & \text{sign}[(\pi_1 - \pi_j)(\pi_{j+1} - \pi_n) - (n-j-1)(\pi_1 - \pi_j)(\pi_n - \pi_{n+1}) \\ & - (\pi_1 - \pi_{j+1})(\pi_j - \pi_n) + (n-j)(\pi_1 - \pi_{j+1})(\pi_n - \pi_{n+1})] \\ = & \text{sign}[(\pi_1 - \pi_j)(\pi_n - \pi_{n+1}) + (n-j)(\pi_j - \pi_{j+1})(\pi_n - \pi_{n+1}) \\ & + (\pi_1 - \pi_j)(\pi_{j+1} - \pi_n) - (\pi_1 - \pi_{j+1})(\pi_j - \pi_n)] \\ = & \text{sign}[(\pi_1 - \pi_j)(\pi_n - \pi_{n+1}) + (n-j)(\pi_j - \pi_{j+1})(\pi_n - \pi_{n+1}) + (\pi_1 - \pi_j)(\pi_{j+1} - \pi_n) \\ & - (\pi_1 - \pi_j)(\pi_j - \pi_n) - (\pi_j - \pi_{j+1})(\pi_j - \pi_n)] \\ = & \text{sign}[(\pi_1 - \pi_j)(\pi_{j+1} - \pi_{n+1}) + (n-j)(\pi_j - \pi_{j+1})(\pi_n - \pi_{n+1}) \\ & - (\pi_1 - \pi_j)(\pi_j - \pi_n) - (\pi_j - \pi_{j+1})(\pi_j - \pi_n)] \end{aligned}$$

However,

$$\pi_j - \pi_n - (\pi_{j+1} - \pi_{n+1}) = \pi_j - \pi_{j+1} - (\pi_n - \pi_{n+1}) > 0$$

and

$$\begin{aligned} (\pi_j - \pi_n) &= \pi_j - \pi_{j+1} + \pi_{j+1} - \pi_{j+2} + \cdots + \pi_{n-1} - \pi_n \\ &> (n-j)(\pi_{n-1} - \pi_n) \\ &> (n-j)(\pi_n - \pi_{n+1}) \end{aligned}$$

Therefore,

$$\lambda_j < \lambda_{j+1}$$

■

### A.7 Proof of Proposition 5

**Proof.** From Lemma 3,  $\lambda_j$  increases in  $j$ . Hence, as the number of firms increases, for any given  $\lambda$ , there exists a  $k(\lambda)$  such that  $\alpha_j(n) > \alpha_j(n+1)$  for all  $j < k$  and  $\alpha_j(n) < \alpha_j(n+1)$  for all  $j \geq k$ . Define  $\tilde{\alpha} = \alpha_k(n+1)$ . Then, if  $\alpha > \tilde{\alpha}$ , we have  $m(n) \geq m(n+1)$  which implies  $T(n) \geq T(n+1)$ . If  $\alpha < \tilde{\alpha}$ , we have  $m(n) < m(n+1)$  which means  $T(n) < T(n+1)$ . ■

### A.8 Proof of Lemma 4

**Proof.** Suppose that  $\hat{\alpha} > \alpha$  but they lead to same equilibrium  $m$ . We have

$$TS(\alpha) = \frac{1}{r}(1 - \theta_{n-m+1})(1 - \theta_{n-m+2}) \cdots (1 - \theta_{n-1})[\alpha S_1 + (1 - \alpha)S_n]$$

and

$$TS(\hat{\alpha}) = \frac{1}{r}(1 - \theta_{n-m+1})(1 - \theta_{n-m+2}) \cdots (1 - \theta_{n-1})[\hat{\alpha} S_1 + (1 - \hat{\alpha})S_n]$$

Hence,

$$\begin{aligned} & TS(\alpha) - TS(\hat{\alpha}) \\ &= \frac{1}{r}(1 - \theta_{n-m+1})(1 - \theta_{n-m+2}) \cdots (1 - \theta_{n-1})(\alpha - \hat{\alpha})(S_1 - S_n) \\ &> 0 \end{aligned}$$

Therefore, total social welfare can be increased by reducing  $\hat{\alpha}$  to  $\alpha$ . ■

### A.9 Proof of Proposition 6

**Proof.** From Proposition 1, firm 1 patents when  $\alpha > \alpha_1$ . When  $\alpha = \alpha_1$ , we have

$$TS(\alpha_1) = \frac{1}{r}[\alpha_1 S_1 + (1 - \alpha_1)S_n]$$

Next consider  $\alpha = \alpha_j$ ,  $j > 1$ . Note that

$$S_1 < S_2 < \cdots < S_n$$

and

$$\alpha S_1 + (1 - \alpha)S_n < S_n$$

(13) becomes

$$\begin{aligned} TS(\alpha_j) &< \frac{1}{r}\theta_{n-1}S_1 \\ &+ \frac{1}{r}\theta_{n-2}(1 - \theta_{n-1})S_n \\ &+ \frac{1}{r}\theta_{n-3}(1 - \theta_{n-2})(1 - \theta_{n-1})S_n \\ &+ \cdots \\ &+ \frac{1}{r}\theta_{n-m+1}(1 - \theta_{n-m+2}) \cdots (1 - \theta_{n-1})S_n \\ &+ \frac{1}{r}(1 - \theta_{n-m+1})(1 - \theta_{n-m+2}) \cdots (1 - \theta_{n-1})S_n \\ &= \frac{1}{r}[\theta_{n-1}S_1 + (1 - \theta_{n-1})S_n] \end{aligned}$$

By (2) and note that  $0 < \theta_{n-1} < 1$ ,

$$\begin{aligned} \alpha_1 &= \frac{\pi_1 - \pi_n}{\frac{\pi_1}{\theta_{n-1}} - \pi_n} \\ &= \frac{\pi_1 - \pi_n}{\pi_1 - \theta_{n-1}\pi_n} \theta_{n-1} \\ &< \theta_{n-1} \end{aligned}$$

Therefore,

$$TS(\alpha_1) > TS(\alpha_j) \quad \text{for } j > 1$$

This completes the proof. ■

## A.10 Proof of Proposition 7

**Proof.** The result immediately follows from Lemma 1, Lemma 4 and Proposition 4. ■

## B Appendix

### B.1 A discussion of finite patent life

Suppose the patent has a finite life and lasts for length  $T_p$ . The expected profit from patenting is

$$\begin{aligned}\Pi'_p &= \int_0^{T_p} [\alpha\pi_1 + (1-\alpha)\pi_n]e^{-rt} dt + \int_{T_p}^{\infty} (1-\alpha)\pi_n e^{-rt} dt \\ &= \delta(T_p) [\alpha\pi_1 + (1-\alpha)\pi_n] + \left[ \frac{1}{r} - \delta(T_p) \right] (1-\alpha)\pi_n\end{aligned}$$

where  $\delta(T_p) = \frac{1-e^{-rT_p}}{r}$ .

The first term is the expected profit during the patent life while the second term is the profit when the patent expires. Note that, as in the basic model,  $\Pi'_p$  is invariant with the rank of discovery,  $j$ .

Let  $\Pi'_s(j)$  be the expected profit if firm  $j$  opts for secrecy given firm  $j+1$  patents. We have

$$\begin{aligned}\Pi'_s(j) &= \int_0^{\infty} \left( \int_0^{T_j} \pi_j e^{-rt} dt \right) (n-j) \lambda e^{-(n-j)\lambda T_j} dT_j \\ &\quad + (1-\alpha) \int_0^{\infty} \left( \int_0^{T_j} \pi_n e^{-rt} dt \right) (n-j) \lambda e^{-(n-j)\lambda T_j} dT_j \\ &\quad + \alpha \int_0^{\infty} \left( \int_{T_j+T_p}^{\infty} \pi_n e^{-rt} dt \right) (n-j) \lambda e^{-(n-j)\lambda T_j} dT_j \\ &= \frac{1}{r} \theta_{n-j} \pi_j + (1-\alpha)(1-\theta_{n-j})\delta(T_p)\pi_n + \alpha \left[ \frac{1}{r} - \frac{1}{r} \theta_{n-j} - (1-\theta_{n-j})\delta(T_p) \right] \pi_n\end{aligned}$$

The first term is the expected profit in period  $j$ . The second term represents the expected profit if the patent applied by firm  $j+1$  is invalid. The last term stands for the profit when the patent is valid and firm  $j$  earns profit only after the patent expires.

Firm  $j$  patents when

$$\Pi'_p > \Pi'_s(j)$$

As in the model with infinite patent life, the above inequality yields a sequence of  $\{\alpha'_j\}$ . However, the results in Lemma 1 do not carry over here.

In particular,  $\{\alpha'_j\}$  may be non-monotonic in  $j$ .

Fortunately, results in Lemma 2 will still hold. Moreover, we will have a weaker version of Proposition 1 as follows:

**Proposition 9** *Suppose the patent has a finite life  $T_p$ . Given  $\alpha$ , there exists a unique  $m'$  such that firm  $m'$  patents while firm  $j$  ( $j < m'$ ) opts for secrecy. Moreover,*

$$m' = \max \{j \mid \Pi'_p > \Pi'_s(j)\}$$

According to the above proposition, to determine the equilibrium  $m'$  one starts from backward searching the first  $j$  that satisfies  $\Pi'_p > \Pi'_s(j)$ .

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