On the Role of Pension Systems in Economic Development and Demographic Transition

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Abstract

In this paper we examine whether different pension systems affect the set of initial human capital conditions capturing an economy in a low steady state equilibrium income. To analyze this problem, we employ a three period overlapping generations model where fertility and investments into the children's education are chosen endogenously. This allows us to show, that education investments are higher and start at lower income levels for a pay-as-you-go pension system compared to an informal, fertility related pension system. The pay-as-you-go income threshold needed to escape the "poverty trap" is therefore lower. Moreover, unless the economy is caught in the poverty trap, equilibrium income in a pay-as-you-go economy is higher. Our model further highlights that pension systems influence the timing of demographic transition, offering a possible explanation for the observed differences between developed and developing countries.

JEL classification: H55, J13, O23

Key words: Pension systems, poverty trap, demographic transition

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1 Introduction

Many countries around the world suffer from persistent underdevelopment. In the year 2001 about 21% of the world population lived below the poverty line of 1$/day (World Bank (2005)). While political and ecological reasons can be responsible for this tragedy we are focusing on economic explanations. A "poverty trap", the economic term for the situation of an economy captured in a low equilibrium per capita income can be caused by a variety of factors like corruption, search externalities (Diamond (1982)), learning-by-doing externalities (Brezis, Krugman, Tsiddon (1993)) or human capital externalities (Azariadis, Drazen (1990)). Our work is focusing on the situation where a demographic trap (Becker, Murphy, Tamura (1990)) is causing a situation where an economy is caught in a vicious circle of low human capital and high population growth which supports low equilibrium income and education. Recent studies show the importance of the amount of pension payments (Boldrin, De Nardi, Jones (2005)) as well as the type of pension system (Holler (2007)) on agents' fertility decisions. We pick up the idea that pension systems play an important role in fertility dynamics aiming to analyze their effect on long-run per capita income and the income threshold needed to escape a poverty trap.

By including a subsistence level of retirement consumption as in Galor and Weil (2000), we reproduce the historically observed inverted U-shaped fertility dynamics corresponding to increasing income levels.[1] This allows us to additionally study the influence of different pension systems on the timing of demographic transition.

Starting point of our analysis is a model discussed by Ken Tabata (2003) which emphasizes the importance of public education investments on human capital accumulation and the possibility of being caught in a poverty trap. In contrast to this paper we focus on the role of different pension systems on demographic transition and the determination of the human capital threshold level needed to approach a high steady state equilibrium income.

To analyze the impacts of a change from one pension system to the other, this paper is comparing equilibrium per capita income and fertility rates corresponding to an informally financed pension system and a pay-as-you-go pension

[1] Works by Dyson and Murphy (1985), Kremer (1993), Lucas (1999) and Lee (2003) give a detailed explanation of fertility reactions along increasing income levels.

system. This enables us to answer the question whether the introduction of a pay-as-you-go pension system can help developing countries to escape a poverty trap. Additionally we can check the viability of a fertility related pension system introduction to diminish a decrease in fertility rates and analyze the corresponding cost.

2 The Model

We assume a standard neoclassical constant returns to scale production sector for a small open economy. The interest rate is exogenously given and constant. Capital is perfectly mobile implying that the capital labor ratio k and the wage rate w are fixed and constant. The Diamond type OLG economy is populated by finitely living homogenous agents. Individuals live for three periods: childhood, adulthood and retirement. During childhood individuals consume $\theta w h_t$, where θ is a fixed fraction of adult working time needed to rear one child and h_t is the amount of human capital an adult at period t is holding. Human capital is determining the effectiveness of labor. Total working income wh_t is therefore increasing in human capital. During adulthood households decide about quantity n_t and quality of children represented by education investments e_t. Education and fertility decisions are implicitly determining the amount of savings s_t. Child quality investments are like quantity investments expressed as working time cost. Following Galor and Weil (2000) we use a Cobb-Douglas utility functions which allows us to abstract from adult consumption without changing the qualitative results. Retirement consumption has to be above a subsistence level $\underline{c} > 0$ which secures surviving when old. The population dynamics for the productive adult population are described by $N_{t+1} = N_t n_t$. Retired people only consume and have no influence on household optimization. They are assumed to use up their whole savings plus pension benefits. Bequests are therefore excluded from the model.

Individuals preferences are defined by retirement consumption above $\underline{c}$ and the future wellbeing of their children. Along the lines of De La Croix and Doepke (2003) adults are therefore drawing utility from the existence and the future human capital of their children which is determining future adult income. Throughout the paper we refer to the second part of the utility function the consumption good value as the altruistic value of child quality and quantity. While the word altruism implies, that actions are made despite any own utility considerations which is not the case in our model we stick to the word to pay tribute to earlier work we are building on.

Individuals utility is represented by the following logarithmic additive separable function:

$$u^t = \beta \log(c_{t+1}) + (1-\beta)\log(n_t h_{t+1}) \tag{1}$$

The discount factor β which is assumed to be smaller than 1 is determining time preference as well as adult altruism towards the children.

Next to the consumption good motive of fertility we additionally model the old age security motive of fertility by incorporating ascending altruism (Wigger (1999)). Following Morand (1999) ascending altruism of individual's preferences is captured through gifts from adult children to their parents during retirement. The ascending altruistic part of preferences is therefore captured in the composition of pension payments π_{t+1}. Inspired by the "intergenerational flow theory" (Caldwell (1982)) we assume that ascending altruism is only present for countries without a mandatory public pension system.[2] Two different pension system scenarios are examined.

Informal pension system: This scenario is describing the situation of developing countries where children are socially responsible for the wellbeing of their retired parents. Pension benefits are therefore dependent on own fertility decisions. Nevertheless the pension contributions are socially mandatory (World Bank (1991)) we use the terminology of ascending altruism to describe the private intrafamilial transfer from adult children to their old parents. The transfers τ are assumed to be lump sum. We further assume that the pension system is always budget balanced implying:

$$\pi^I_{t+1} = n^I_t \tau$$

Pay-as-you-go pension system: This case examines economies with a functioning public mandatory pay-as-you-go pension system. In the absence of bequests old age support is the only motivation for private interfamilial gifts.[3] If the state takes over the role of supporting the old generation private gifts are therefore fully crowded out. A mandatory public pension system further

[2] A more detailed description of this argument is provided in Holler (2007).

[3] Positive bequests would lead to children contributing to their parents pensions through private gifts in expectation of bequests from their parents at the end of their lives. For a detailed description of the different bequest motives see Zhang and Zhang (2001) or (WB 1991).

implies that pension payments do not depend on individual fertility but on average fertility of the whole economy $\overline{n}_t$.

$$\pi^P_{t+1} = \overline{n}^P_t \tau$$

Adults endowed with a human capital level h_t divide their after tax income $h_t w - \tau$ between child cost (rearing cost $\theta n_t h_t w$ + child education cost $e_t n_t h_t w$) and savings since they do not draw utility from consumption when adult. The adulthood budget constrained is therefore given by:

$$w h_t n_t (\theta + e_t) + \tau + s_t \leqslant w h_t \tag{2}$$

Retirement consumption is financed through the value of savings at period $t+1$ plus pension benefits. Agents consume their whole retirement income since we assumed that bequests are zero. Following Galor and Weil (2000) minimum retirement consumption is limited by a subsistence level $\underline{c}$.

$$c_{t+1} = R s_t + \pi_{t+1} \tag{3}$$

$$c_{t+1} \geqslant \underline{c} \tag{4}$$

Economic growth is solely determined by the evolution of human capital over time. Following Tabata (2006) human capital accumulation is determined by education investments, adult human capital level h_t and productivity of the education sector determined by the parameters a, b, η and σ.

$$h_{t+1} = \eta (a + b e_t h_t)^{\sigma}; \eta, a, b > 0; 0 < \sigma < 1 \tag{5}$$

Adult human capital is entering the accumulation formula to capture the positive influence of parental human capital on the child's future skills. The positive a parameter is securing that future human capital is positive in the case of zero education investments. Since σ is smaller than one each additional unit of education investment pays less in terms of additional future human capital.

Equation (1) subject to (2), (3), (4) and (5) describe the household optimization problem. For sufficiently high income supporting consumption above the subsistence level the optimization leads to first order conditions 1 to 4. Superscript P and I specify pay-as-you-go and informal pension system variables.

$$e_t^I : \beta \frac{wh_t R n_t^I}{c_{t+1}^I} = (1-\beta)\frac{bh_t\sigma}{a + be_t^I h_t} \tag{FOC 1}$$

$$n_t^I : \beta \frac{wh_t R(\theta + e_t^I)}{c_{t+1}^I} = (1-\beta)\frac{1}{n_t^I} + \beta\frac{\tau}{c_{t+1}^I} \tag{FOC 2}$$

$$e_t^P : \beta \frac{wh_t R n_t^P}{c_{t+1}^P} = (1-\beta)\frac{bh_t\sigma}{a + be_t^P h_t} \tag{FOC 3}$$

$$n_t^P : \beta \frac{wh_t R(\theta + e_t^P)}{c_{t+1}^P} = (1-\beta)\frac{1}{n_t^P} \tag{FOC 4}$$

Adults can either invest in child quantity (n_t) or child quality (e_t). At the optimum, marginal benefit of the investments have to equal marginal cost. FOC 1 and 3, describing optimal education decisions for both pension systems, state that the marginal value of education measured in terms of additional future human capital has to equal marginal cost of education measured in terms of retirement consumption. In other words at the point where marginal altruistic utility of additional future child income equals marginal cost of reduced retirement consumption education investments are optimal. Optimal Fertility decisions covered in FOC 2 and FOC 4 demand that marginal cost of a child are equal to marginal benefits. FOC 2 further shows that marginal child utility is split into an altruistic and a retirement consumption part for the informal pension system. This is due to the existence of positive intrafamilial gifts. For an economy with a pay-as-you-go pension system marginal child utility is solely determined by altruism (see FOC 4). The first order conditions further highlight that for both pension system cases a quality quantity trade-off a la Becker and Barro (1988) is in place. High investments in child education are implying low fertility and vice versa.

After describing the situation of relatively high income levels, supporting retirement consumption above or equal to the subsistence level, we focus towards the low income cases. If income levels can not support subsistence, retirement consumption condition (4) becomes binding and optimal decisions are described by FOC 5.

$$\underline{c} = Rwh_t - (wh_t R(\theta + e_t) - \tau)n_t - \tau R \tag{FOC 5}$$

2.1 Education

In the described model education investments are solely driven by altruism because they do not create any benefit in the form of retirement consumption. Therefore we assume parents to choose positive education investments only if parental income is supporting a retirement consumption level above subsistence. In other words investments in the quality of children only take place if old age survival is secured. The parameter assumptions connected to this assumption are summarized in Lemma 1.

Lemma 1 *If $a > \frac{(R\beta\tau + \underline{c})b\theta\sigma}{Rw\beta}$ the human capital level supporting the subsistence level of consumption $\underline{h}$ is lower than the human capital level that supports positive education investments $\overline{h}$ for both pension systems.*

To make things easier we skip the proof of Lemma 1 to subsection 2.3. Due to our parameter assumptions we can observe two optimal education results depending on whether income is below or above the positive education investment threshold $\overline{h}$.

$$e_t^P = \begin{cases} 0 & \text{if } h_t \leqslant \overline{h}^P \\ \frac{b\theta\sigma h_t - a}{b(1-\sigma)h_t} & \text{if } h_t \geqslant \overline{h}^P = \frac{a}{b\theta\sigma} \end{cases}$$

$$e_t^I = \begin{cases} 0 & \text{if } h_t \leqslant \overline{h}^I \\ \frac{bRw\theta\sigma h_t - aRw - b\sigma\tau}{b(1-\sigma)Rwh_t} & \text{if } h_t \geqslant \overline{h}^I = \frac{aRw + b\sigma\tau}{bRw\theta\sigma} \end{cases}$$

From optimal education decisions we follow that the threshold level needed to make education decisions positive is different for both pension systems.

Proposition 1 *The positive education threshold $\overline{h}$ is higher for the informal pension system case ($\overline{h}^I > \overline{h}^P$) implying that it takes higher income levels to make education investments positive. From $\overline{h}^P$ onwards pay-as-you-go education investments are higher than informal ones.*

Proof. $\overline{h}^I = \overline{h}^P + \underbrace{\frac{\tau}{Rw\theta}}_{>0}; e_t^P = e^I + \underbrace{\frac{\sigma\tau}{(1-\sigma)Rwh_t}}_{>0}.$ ■

2.2 Fertility

Optimal fertility decisions are again dependent on the level of adult income. Due to our parameter assumptions we have to differentiate between the following three cases. The first case of human capital below the subsistence threshold

$\underline{h}$ describes the situation where the subsistence retirement consumption assumption is binding and education investments are zero. $\underline{h} \leqslant h_t \leqslant \overline{h}$ corresponds to the second case where investments in child quality are still zero but the income level is already high enough to lead to retirement consumption above the subsistence level. The third case $h_t \geqslant \overline{h}$ is reflecting the situation of relatively high human capital supporting positive education investments.

Use the fact that average fertility is equal to individual fertility because agents are homogenous to get optimal pay-as-you-go fertility decisions:

$$n_t^P = \begin{cases} \frac{\underline{c}+R\tau-Rwh_t}{\tau-Rw\theta h_t} & \text{if } h_t \leqslant \underline{h}^P \\ \frac{R(\beta-1)(\tau-wh_t)}{(\beta-1)\tau+Rw\theta h_t} & \text{if } \underline{h}^P \leqslant h_t \leqslant \overline{h}^P \\ \frac{bR(\beta-1)(\sigma-1)(\tau-wh_t)}{aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

Optimal informal fertility decisions are represented by:

$$n_t^I = \begin{cases} \frac{\underline{c}+R\tau-Rwh_t}{\tau-Rw\theta h_t} & \text{if } h_t \leqslant \underline{h}^I \\ \frac{R(\beta-1)(\tau-wh_t)}{Rw\theta h_t-\tau} & \text{if } \underline{h}^I \leqslant h_t \leqslant \overline{h}^I \\ \frac{bR(1-\beta)(\sigma-1)(\tau-wh_t)}{bRw\theta h_t-aRw-b\tau} & \text{if } h_t \geqslant \overline{h}^I \end{cases}$$

From equation (5) and Proposition 1 we know that for income levels $h_t < \overline{h}^P$ human capital is constant at ηa^σ. This enables us to directly compare optimal fertility decisions for this income range. We follow that informal and pay-as-you-go fertility are equal for income levels below the subsistence threshold ($h_t < \underline{h}$). Optimal fertility results further imply that the income level needed to surpass minimum retirement consumption is different for both pension systems.

$$\underline{h}^I = \frac{R\beta\tau + \underline{c}}{Rw\beta}$$

$$\underline{h}^P = \frac{\underline{c}Rw\theta + R^2w\beta\theta\tau \pm \sqrt{4\underline{c}R^2w^2(\beta-1)\beta\theta\tau + (\underline{c}Rw\theta + R^2w\beta\theta\tau)^2}}{2R^2w^2\beta\theta}$$

As long as income is below the threshold $\underline{h}$ retirement consumption is constant at $\underline{c}$. In this case parents would like to give up retirement consumption in order to have more children. This is not possible because retirement consumption is at a level needed for survival and condition (4) becomes binding. Implicitly the existence of this case demands that children are a costly investment. In other words opportunity cost of children have to be higher than benefits ($Rw\theta h_0 > \tau$). If this would not be the case condition (4) could never become binding because having more children would not decrease but increase retirement consumption. The assumption that retirement consumption has to be high enough to secure survival together with the fact that fertility has to be positive limits initial human capital to:

$$h_0 > \max\left\{\frac{\tau}{Rw\theta}, \frac{\underline{c} + R\tau}{Rw}\right\}$$

We allow fertility for the lowest possible income level to be smaller than one. In these cases adults can only secure old age survival by choosing fertility rates less than 1. Our model therefore also captures income cases corresponding to shrinking adult population due to a lack of resources. $n_0 < 1$ is only possible if $0 < \underline{c}\theta + (R\theta - 1)\tau$.[4] This changes the minimum human capital condition to:

$$h_0 > \frac{\underline{c} + R\tau}{Rw}$$

As income increases and surpasses $\underline{h}$ agents enjoy retirement consumption above the subsistence level. Education investments are still zero ($\underline{h}^P < h_t < \overline{h}^P$). In this income range individuals use resources above the subsistence level not only to have children but also to increase retirement consumption through higher savings. Households therefore weight the marginal benefit of children which now also includes altruistic utility against the marginal benefit of additional consumption through higher savings (FOC 2 and FOC 4 with $e_t = 0$). Both pension systems still face the same level of human capital. Comparing optimal decisions highlights that pay-as-you-go fertility is smaller than informal fertility.

If the income level is high enough ($h_t > \overline{h}$) human capital starts to grow due to positive education investments. The income level needed to impose positive education investments and the amount of education investments is different for the two pension systems. From $\overline{h}^P$ onwards pay-as-you-go human capital is higher than informal human capital. This is the reason why a simple comparison of informal and pay-as-you-go fertility decisions can not be performed for the high income case. We skip this exercise to section 3 which focuses on a detailed examination of fertility dynamics.

[4] If $0 \geqslant \underline{c}\theta + (R\theta - 1)\tau : h_t > \frac{\tau}{Rw\theta} \geqslant \frac{\underline{c}+R\tau}{Rw}$. Reformulation gives us $h_t - \frac{\tau}{Rw\theta} \leqslant h_t - \frac{\underline{c}+R\tau}{Rw}$ and $Rw\theta h_t - \tau \leqslant \theta(Rwh_t - \underline{c} - R\tau) < Rwh_t - \underline{c} - R\tau$. This shows that $n_t = \frac{\underline{c}+R\tau-Rwh_t}{\tau-Rw\theta h_t}$ can only be smaller than 1 if $0 < \underline{c}\theta + (R\theta - 1)\tau$.

2.3 Consumption

The budget constraints (2) and (3) together with optimal education and optimal fertility decision determines retirement consumption.

$$c_{t+1}^{I}=\begin{cases}\underline{c} & \text{if } h_t \leqslant \underline{h}^{I}\\ R\beta(wh_t-\tau) & \text{if } h_t \geqslant \underline{h}^{I}\end{cases}$$

$$c_{t+1}^{P}=\begin{cases}\underline{c} & \text{if } h_t \leqslant \underline{h}^{P}\\ \frac{R^2 w\beta\theta h_t(wh_t-\tau)}{(\beta-1)\tau+Rw\theta h_t} & \text{if } \overline{h}^{P} \geqslant h_t \geqslant \underline{h}^{P}\\ \frac{R^2 w\beta(\tau-wh_t)(-a+b\theta h_t)}{aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^{P}\end{cases}$$

Proposition 2 *A pay-as-you-go pension system economy demands lower income levels to support consumption above a subsistence level than an informal pension system economy* ($\underline{h}^{P}<\underline{h}^{I}$).

Proof. Assume $\underline{h}^{I} \leqslant \underline{h}^{P}$ and $\underline{h}=\underline{h}^{P}$. Informal retirement consumption is therefore equal or bigger than subsistence retirement consumption ($c_{t+1}^{I}=R\beta(w\underline{h}-\tau)\geqslant \underline{c}$) and pay-as-you-go retirement consumption is equal to retirement consumption ($c_{t+1}^{P}=\frac{R^2 w\beta\theta\underline{h}(w\underline{h}-\tau)}{(\beta-1)\tau+Rw\theta\underline{h}}=\underline{c}$). It follows that $R\beta(w\underline{h}-\tau)\geqslant \frac{R^2 w\beta\theta\underline{h}(w\underline{h}-\tau)}{(\beta-1)\tau+Rw\theta\underline{h}}$. Reformulation gives us $1\geqslant\frac{Rw\theta\underline{h}}{(\beta-1)\tau+Rw\theta\underline{h}}$. Because the right hand side of this expression is bigger than 1, $\underline{h}^{I}\leqslant\underline{h}^{P}$ can not be true, proofing that $\underline{h}^{I}>\underline{h}^{P}$. ■

At low income levels retirement consumption is constant at the subsistence level. As a certain income threshold is surpassed individuals start to increase consumption. Lower income levels are needed to increase pay-as-you-go retirement consumption above $\underline{c}$ than in the informal case. The result is driven by the fact that marginal benefit of having a child is lower for the pay-as-you-go pension system since pension benefits are independent on own fertility decisions. Therefore for each income level the demand for children is lower than in the informal pension system making it easier for retirement consumption to increase a subsistence level.

Next to the different threshold, all income levels above $\underline{h}^{P}$ retirement consumption support higher retirement consumption for a pay-as-you-go pension system economy. This is the case because savings are higher due to lower fertility investments.

Through the help of the already derived insights we are now in the position to proof Lemma 1 which is securing that education investments only take place if income surpasses the subsistence level of consumption.

Proof of Lemma 1. $\underline{h} \leqslant \overline{h}$ demands that $\underline{h}^I \leqslant \overline{h}^I$ and $\underline{h}^P \leqslant \overline{h}^P$. Because $\underline{h}^I > \underline{h}^P$ and $\overline{h}^I > \overline{h}^P$, $\underline{h} \leqslant \overline{h}$ is true if $\underline{h}^I < \overline{h}^P$. Now plug in $\underline{h}^I = \frac{R\beta\tau+\underline{c}}{Rw\beta}$ and $\overline{h}^P = \frac{a}{b\theta\sigma}$ to see that this is the case if $a > \frac{(R\beta\tau+\underline{c})b\theta\sigma}{Rw\beta}$. ■

2.4 Savings

Education investment- and fertility decisions are fully describing the behavior of savings. Nevertheless in order to completely describe the model we produce the following results for optimal savings:

$$s_t^P = \begin{cases} \frac{\tau^2+(-w\tau+w\theta\underline{c})h_t}{Rw\theta h_t-\tau} & \text{if } h_t \leqslant \underline{h}^P \\ \frac{(wh_t-\tau)((\beta-1)\tau+Rw\beta\theta h_t)}{(\beta-1)\tau+Rw\theta h_t} & \text{if } \underline{h}^P \leqslant h_t \leqslant \overline{h}^P \\ \frac{(wh_t-\tau)(aRw\beta+b(\beta-1)(\sigma-1)\tau-bRw\beta\theta h_t)}{aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

$$s_t^I = \begin{cases} \frac{\tau^2+(-w\tau+w\theta\underline{c})h_t}{Rw\theta h_t-\tau} & \text{if } h_t \leqslant \underline{h}^I \\ \frac{(wh_t-\tau)(\tau-Rw\beta\theta h_t)}{\tau-Rw\theta h_t} & \text{if } \underline{h}^I \leqslant h_t \leqslant \overline{h}^I \\ \frac{(wh_t-\tau)(aRw\beta+b(1+(\beta-1)\sigma)\tau-bRw\beta\theta h_t)}{aRw+b\tau-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^I \end{cases}$$

Corresponding to fertility decisions informal and pay-as-you-go savings are equal for income levels below the pay-as-you-go subsistence threshold $\underline{h}^P$ and lower for the range $\underline{h}^P < h_t < \overline{h}^P$ because informal fertility is higher. Due to the fact that h_t is different for human capital levels bigger than $\overline{h}^P$ a simple direct comparison of the optimal decisions for this high income range can not be performed.

2.5 Human Capital Accumulation and the Poverty Trap

Now we are focusing towards the differences in human capital accumulation due to differences in education investments for the two pension systems. Equation (5) and optimal education decisions determine the human capital accumulation equations:

$$h_{t+1}^P = \begin{cases} \eta a^\sigma \equiv \Gamma_1(h_t) & \text{if } h_t \leqslant \overline{h}^P \\ \eta\left(a + \frac{b\theta\sigma h_t - a}{(1-\sigma)}\right)^\sigma \equiv \Gamma_2^P(h_t) & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

$$h_{t+1}^{I}=\begin{cases}\eta a^{\sigma}\equiv\Gamma_1(h_t) & \text{if } h_t\leqslant\overline{h}^{I}\\ \eta\left(a+\frac{-aRw-b\sigma\tau+bRw\theta\sigma h_t}{(1-\sigma)Rw}\right)^{\sigma}\equiv\Gamma_2^{I}(h_t) & \text{if } h_t\geqslant\overline{h}^{I}\end{cases}$$

Where $\Gamma_1(h_t)$ is a line and $\Gamma_2(h_t)$ is a concave function ($\frac{\partial\Gamma_2(h_t)}{\partial h_t}>0$, $\frac{\partial^2\Gamma_2(h_t)}{\partial h_t^2}<0$).

Depending on the parameter values the described model has different steady state equilibria. We are focusing on the case of a poverty trap[5] since we want to observe whether the different pension systems have an influence on the income level needed to escape the low long-run per capita income equilibrium.

Proposition 3 *If* $\min\left\{\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{(\sigma-1)\sigma}}+b\left(\eta\theta\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}-\frac{\tau}{Rw}\right),\ b\sigma\left(\eta\theta\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}-\frac{\tau}{Rw}\right)\right\}>a>(b\theta\sigma\eta)^{\frac{1}{1-\sigma}}$ *the model generates two stable and one instable steady state equilibria for both pension systems. Initial income lower than the poverty trap threshold* $\overline{h}_{trap}^{P}$ *leads to steady state equilibria that equal each other for both pension systems. If initial human capital levels are higher or equal to* $\overline{h}_{trap}^{P}$ *a pay-as-you-go public pension system supports a higher steady state equilibrium than the informal pension system.*

Proof. The assumption $a>(b\theta\sigma\eta)^{\frac{1}{1-\sigma}}$ implying that $\eta a^{\sigma}<\overline{h}^{P}$ secures that a stable low income steady state (E_1) exists for both pension system cases because $\Gamma_1(h)$ intersects the 45° line. Now rearrange $\Gamma_2^{I}(h)=h$ to separate a linear and a power function. This gives us: $\underbrace{bRw\theta h}_{L(h)}=\underbrace{\left(\frac{h}{\eta}\right)^{\frac{1}{\sigma}}Rw\frac{1-\sigma}{\sigma}+aRw+b\tau}_{R(h)}$. The value $h^{'}$ that equals the slopes of the two functions $(L^{'}(h^{'})=R^{'}(h^{'}))$ is given by $\eta\left(\frac{1-\sigma}{b\theta\eta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}$. Now compare the functional value of the two curves at $h^{'}$. If the functional value of the power function is lower than the functional value of the line $R(h)$ intersects $L(h)$ twice because $R(0)>0$ and $\frac{\partial R(h)}{\partial h}>0$. These two intersects are also solutions to $\Gamma_2^{I}(h)=h$ implying that $\Gamma_2^{I}(h)$ has two intersects with the 45° line. $R(h^{'})=bRw\theta\eta\left(\frac{1-\sigma}{b\theta\eta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}$; $L(h^{'})=Rw\,(1-\sigma)\left[\left(\frac{1-\sigma}{b\theta\eta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1}{\sigma}}+aRw+b\tau$. $R(h^{'})<L(h^{'})$ if $\frac{(\sigma-1)}{\sigma}\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{(\sigma-1)\sigma}}+b\left(\eta\theta\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}-\frac{\tau}{Rw}\right)>a$. The lower steady state equilibrium (E_2^{I}) is unstable, the higher one is stable (E_3^{I}). Pay-as-you-go education investments are always higher than informal

[5] Situation where at least two stable (one low and one high) and one unstable steady states exist. The variable value supporting the unstable equilibrium forms a threshold in reaching the high stable steady state equilibrium.

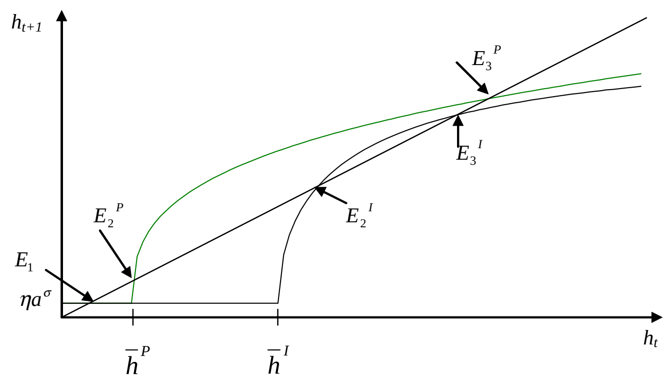
ones. Therefore the described parameter restrictions also produce an unstable steady state equilibrium (E_2^P) and a stable steady state equilibrium (E_3^P) for the pay-as-you-go pension system. The last step of the proof secures the existence of E_2 and E_3. In order to exist, the corresponding human capital levels have to be bigger than the threshold $\overline{h}$. This is the case if $h^{'} > \overline{h}^I$ and $\eta a^{\sigma} < \overline{h}^P$. $h^{'} > \overline{h}^I$ if $a < b\sigma \left(\eta\theta \left(\frac{1-\sigma}{b\eta\theta\sigma^2} \right)^{\frac{\sigma}{\sigma-1}} - \frac{\tau}{Rw} \right)$. ■

Proposition 4 *A pay-as-you-go pension system economy is featured by a smaller set of initial income conditions supporting a poverty trap equilibrium because the human capital threshold needed to approach the high steady state equilibrium is lower* ($\overline{h}_{trap}^P < \overline{h}_{trap}^I$).

As long as income is below the education threshold $\overline{h}^P$ both pension systems have an identical steady state equilibrium at $h^* = \eta a^{\sigma}$. From $\overline{h}^P$ onwards pay-as-you-go human capital is bigger than informal human capital because education investments are always higher in the pay-as-you-go case (see Proposition 1). The unstable steady state equilibrium determining the poverty trap threshold level ($\overline{h}_{trap}$) is therefore lower for the pay-as-you-go case ($E_2^P < E_2^I$). Additionally the stable pay-as you go positive education steady state is higher $E_3^P > E_3^I$. It follows that lower initial human capital is needed ($\overline{h}_{trap}^P < \overline{h}_{trap}^I$) to approach an even higher stable steady state in the case of a pay-as-you-go pension system.

Figure 1 fully describes the behavior of both human capital accumulation equations and the corresponding equilibria.

Figure 1: Human Capital Accumulation



As long as education investments are zero future human capital of both pension systems is constant and equal ($E_1^P = E_1^I$). As income determining human capital reaches the positive education threshold ($\bar{h}$) which is lower for the pay-as-you-go case, future human capital starts to rise. If initial human capital is below (above) the level $\bar{h}_{trap}$ which is corresponding to E_2 the economy approaches the stable low (high) steady state equilibrium $E_1(E_3)$. Throughout the literature initial human capital lower than $\bar{h}_{trap}$ is known as a "poverty trap" scenario.

Besides the described 3 steady state poverty trap scenario different parameter values support a variety of equilibria[6]. While we do not examine each case in detail, one can state that all cases with different stable equilibria support a lower poverty trap threshold for the pay-as-you-go pension system. All income levels above this threshold lead to higher long-run per capita for the pay-as-you-go pension system. This is the case because education investments start at lower income levels and are always higher in the pay-as-you-go case because marginal utility of procreation is lower.

Proposition 5 *For all possible parameter values supporting different stable equilibria the set of initial conditions leading to a stable high equilibrium is larger*

[6] Examples of further existing cases can be examined in Apendix A.

for an economy with pay-as-you-go pension system compared to an economy with an informally financed pension system. All parameter values corresponding to a stable steady state equilibrium which is different to $h^* = \eta a^\sigma$ lead to higher long-run per capita income for the pay-as-you-go case. Only if ηa^σ is the unique stable steady state equilibrium both pension systems imply the same long-run per capita income.

After the comparison of equilibria and connected thresholds we focus on the role of pension systems for demographic transition.

3 Demographic Transition

While the term demographic transition describes the behavior of fertility and mortality over time we are only focusing on fertility dynamics. Economic studies usually substitute time for income because historically income is increasing over time. One can observe that income and fertility are positively related for low income regions while the opposite is true for high income regions [7]. In order to enable our model to cover the empirical fact that fertility rates are negatively dependent on income increases for high income levels we assume that parameters satisfy $R\theta + \beta + \sigma < 1 + \beta\sigma$. This additional parameter assumption enables our model to nicely cover the pattern of historic fertility dynamics for the three income cases $h_t \leqslant \underline{h}, \underline{h} \leqslant h_t \leqslant \overline{h}$ and $h_t \geqslant \overline{h}$.

3.1 Malthusian State: $h_t \leqslant \underline{h}$

This, low income scenario describes the situation where the subsistence level of retirement consumption assumption is binding. Because individuals primarily have to secure their survival and use income above the subsistence level only to procreate, education investments are zero. The term Malthusian state is nicely describing this situation because additional income is directly translated to higher fertility while consumption per capita stays constant. To proof that this is the case, take the first derivative of fertility with respect to human capital.

$$\frac{\partial n_t^P}{\partial h_t} = \frac{\partial n_t^I}{\partial h_t} = \frac{Rw\left(\left(R\theta - 1\right)\tau + \theta \underline{c}\right)}{\left(\tau - Rw\theta h_t\right)^2} > 0$$

[7] For a detailed description of the Demographic Transition see for example Lee 2003.

The second derivative highlights that fertility increases due to income increases take place at a decreasing rate.

$$\frac{\partial^2 n_t^P}{\partial h_t^2} = \frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{2R^2 w^2 \theta \left((R\theta - 1)\, \tau + \theta \underline{c} \right)}{\left(\tau - Rw\theta h_t\right)^3} < 0$$

Proof. As already explained, initial fertility is assumed to be lower than 1 implying that $\underline{c}\theta + (R\theta - 1)\tau > 0$. Therefore $\frac{\partial n_t}{\partial h_t} > 0$ and $\frac{\partial^2 n_t}{\partial h_t^2} < 0$. ■

For low income levels our model is reproducing the Malthusian view of an economy that can not prosper because income increases are only used for additional procreation. As already sketched fertility is equal for both pension systems if retirement consumption is fixed at $\underline{c}$. In this economic stage only the threshold level of income needed to support consumption above the subsistence level is depending on the pension system. A lower level of income is needed to induce additional savings for the pay-as-you-go pension system due to the fact that marginal benefit of fertility is lower. This is translating to lower fertility and higher savings enabling consumption to surpass the subsistence level at a lower income level.

3.2 Post-Malthusian State: $\overline{h} \geq h_t \geq \underline{h}$

Now human capital becomes sufficient to support retirement income above the subsistence level. Agents still do not contribute working time to educate their children. Different to the Malthusian state, income increases lead to lower fertility rates because the alternative investment opportunity of additional retirement consumption through higher savings reduces the demand for children. To see that this is true calculate the first order derivative of fertility with respect to human capital:

$$\frac{\partial n_t^P}{\partial h_t} = \frac{Rw(1-\beta)\,(\beta + R\theta - 1)\,\tau}{\left((\beta - 1)\tau + Rw\theta h_t\right)^2} < 0; \frac{\partial n_t^I}{\partial h_t} = \frac{Rw(1-\beta)(R\theta - 1)\tau}{\left(\tau - Rw\theta h_t\right)^2} < 0$$

The second derivative further show that for both pension systems the decrease in fertility connected to increasing income takes place at a decreasing rate.

$$\frac{\partial^2 n_t^P}{\partial h_t^2} = \frac{2R^2 w^2 \theta (\beta - 1)\,(\beta + R\theta - 1)\,\tau}{\left((\beta - 1)\tau + Rw\theta h_t\right)^3} > 0; \frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{2R^2 w^2 \theta (1-\beta)\,(R\theta - 1)\,\tau}{\left(\tau - Rw\theta h_t\right)^3} > 0$$

As already mentioned an economy with pay-as-you-go pension system enters this Post-Malthusian state at lower income levels than economies with informal pension system. This is the case because children pay less in the pay-as-you-go system leading to higher opportunity cost of not investing in savings. Savings are therefore higher and the demand for children is lower. Pay-as-you-go retirement consumption exceeds informal retirement consumption because savings pay more than fertility. The combination of the Malthusian and Post-Malthusian state without education investments already sketches the main features of demographic transition. Income increases lead to increasing (decreasing) fertility for low (high) income regions.

3.3 Post-Malthusian State with positive education investments: $h_t \geqslant \overline{h}$

Now education investments become positive. In this stage of economic development fertility is not only competing against additional retirement consumption but also against investments in the quality of children. The correlation between income and fertility is still negative.

$$\frac{\partial n_t^P}{\partial h_t} = \frac{bRw(\beta-1)(\sigma-1)(-aRw+b(\beta+R\theta+\sigma-\beta\sigma-1)\tau)}{(aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t)^2} < 0$$

$$\frac{\partial n_t^I}{\partial h_t} = \frac{bRw(\beta-1)(\sigma-1)(-aRw+b(R\theta-1)\tau)}{(aRw+b\tau-bRw\theta h_t)^2} < 0$$

The decrease in fertility due to increasing income is again decreasing.

$$\frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{2b^2R^2w^2\theta(\beta-1)(\sigma-1)(-aRw+b(R\theta-1)\tau)}{(aRw+b\tau-bRw\theta h_t)^3} > 0$$

Proof. $h_t = \frac{aRw+b\sigma\tau}{bRw\theta\sigma} : \frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{\overbrace{2b^2R^2w^2\theta(\beta-1)(\sigma-1)(-aRw+b(R\theta-1)\tau)}^{<0}}{\underbrace{(aRw-\frac{aRw}{\sigma})^3}_{<0 \text{ because } \sigma<1}}.$

■

$$\frac{\partial^2 n_t^P}{\partial h_t^2} = \frac{2b^2R^2w^2\theta(\beta-1)(\sigma-1)(-aRw+b(\beta+R\theta+\sigma-\beta\sigma-1)\tau)}{(aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t)^3} > 0$$

Proof. Plug in $h_t = \frac{a}{b\theta\sigma}$ to get $\frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{\overbrace{2b^2R^2w^2\theta(\beta-1)(\sigma-1)(-aRw+b(R\theta-1)\tau)}^{<0}}{(aRw+b(\beta-1)(\sigma-1)\tau - Rw\frac{a}{\sigma})^3}$. The numerator of the expression is negative implying that the second derivative is positive if $a > \frac{b\sigma\tau(1-\beta)}{Rw}$. From the minimum initial human capital constraint we can follow that $\overline{h}^P > \frac{\tau}{Rw\theta}$. Rewritten this condition to get $a > \frac{b\sigma\tau}{Rw}$. Therefore $a > \frac{b\sigma\tau(1-\beta)}{Rw}$ has to be true and the second derivative is positive. ■

As already discussed positive education threshold levels are depending on the pension system. Positive pay-as-you go education investments are supported by lower human capital levels than informal education investments. The lower threshold is again based on the lower marginal benefit of a child in the pay-as-you-go pension system making it easier for education investments to compete. If the threshold is surpassed, higher income leads to higher education investments at a decreasing rate.

$$\frac{\partial e_t^P}{\partial h_t} = \frac{a}{b(1-\sigma)h_t^2} > 0; \frac{\partial^2 e_t^P}{\partial h_t^2} = -\frac{2a}{b(1-\sigma)h_t^3} < 0$$

$$\frac{\partial e_t^I}{\partial h_t} = \frac{aRw + b\sigma\tau}{bRwh_t^2(1-\sigma)} > 0; \frac{\partial^2 e_t^I}{\partial h_t^2} = -\frac{2(aRw + b\sigma\tau)}{bRwh_t^3(1-\sigma)} < 0$$

For all income levels above the pay-as-you-go education threshold agents allocate more time to child quality in the pay-as-you-go pension system case. As income is increasing the difference in education investments of the two pension system cases is decreasing because $\frac{\partial e_t^P}{\partial h_t} < \frac{\partial e_t^I}{\partial h_t}$.

Now we combine the derived insights of the three states to compare informal and pay-as-you-go fertility for all income levels.

Proposition 6 *Pay-as-you-go fertility is equal to informal fertility for all income levels below the pay-as-you-go subsistence threshold* $\underline{h}^P$*. At the income level* $\underline{h}$ *both pension systems reach their fertility maximum. Pay-as-you-go fertility is lower than informal fertility for income levels above* $\underline{h}^P$*.*

The three analyzed income cases also cover information about the observed differences in the timing of demographic transition between developing and developed countries.

Proposition 7 *The introduction of a mandatory pay-as-you-go public pension system to a country with informal, fertility related pension system shifts down the inverted U-shaped demand for children connected to income increases. There-*

fore lower levels of income support an escape of the first stage of demographic transition where income increases lead to increasing fertility.

Lucas (2002) shows that while the demographic transition in the USA and Western Europe already started at the end of the 19^{th} century it took until the 1950s to start in the African Countries. Besides the introduction of a public pension system that first took place in 1889 in Germany of course a lot of other factors connected to the industrial revolution like mortality declines play a role in explaining the different timing of demographic transition. Nevertheless the existence of public pension systems appears to play a significant role.

Our model also suggests that developing countries aiming to reduce population growth should introduce a pay-as-you-go pension system. This fertility demand reduction is additionally accompanied by a lower income threshold needed to escape a poverty trap equilibrium (see proposition 4).

Proposition 8 *Post-Malthusian income levels support a trade-off between fertility and per capita income. Therefore a shift from a pay-as-you-go public pension system to a fertility related informal pension system increases fertility rates but decreases long-run per capita income.*

Proposition 8 suggests that countries experiencing strong fertility declines due to income increases can weaken the effect, by switching to fertility related pension systems. While such a policy will increase the demand for children, long-run per capita income will decrease, highlighting the existing trade-off between fertility and income.

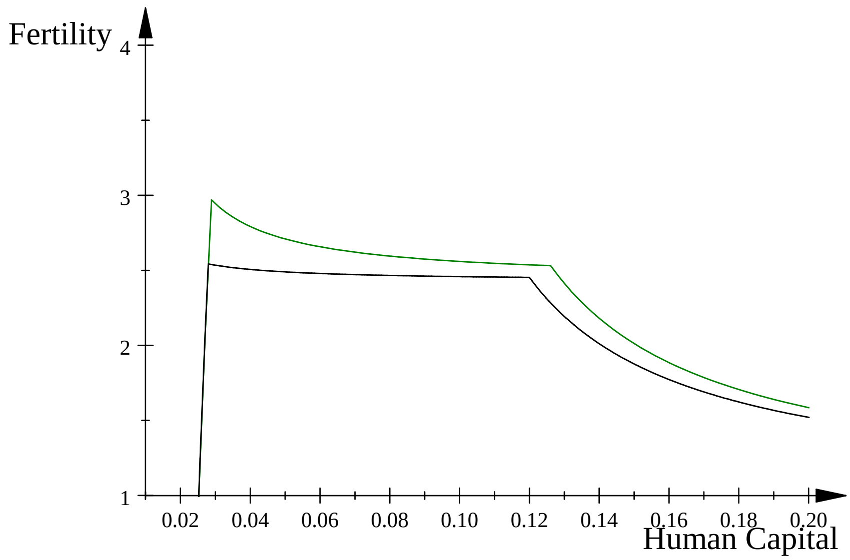
The following subsection presents a numerical example to additionally clarify and summarize fertility dynamics.

3.4 Numerical example

The last subsection already produced all necessary insights to compare the fertility rates of the two pension systems for all development stages. Additionally, we are already able to describe the behavior of fertility due to income increases. The last outstanding aspect to fully sketch fertility dynamics is the difference in the strength of fertility reductions for the two Post-Malthusian stages. Because an analytic comparison of the partial derivatives does not produce a clear result, a numerical example is performed.

Parameter values are set to satisfy the parameter conditions of a multiple equilibria case (see Proposition 3).[8]

Figure 2: Fertility dynamics



The numeric example highlights that fertility dynamics show an acceleration of fertility decrease if education investments become positive. This supports the intuition that an additional competitive investment opportunity further reduces the demand for children.

4 Conclusion

We show that economic development and population dynamics are dependent on the type of pension system. A pay-as-you-go pension system implies lower marginal benefit of fertility compared to an informal, fertility related pension system. The demand for children is therefore lower. Reflecting the quantity quality trade-off pay-as-you-go education investments are higher and start at lower income levels. This is the reason why economic take-off to a high long-run

[8]

a	b	w	R	$\underline{c}$	β	θ	η	σ	τ
0.027	5	100	1.04^{20}	1.6	0.99^{20}	0.075	1	0.6	0.1

equilibrium per capita income is supported by lower income levels for a pay-as-you-go pension system economy. Next to the lower poverty trap threshold, pay-as-you-go education investments which exceed informal ones imply larger per capita income for the high steady state equilibrium.

A switch from an informal- to a pay-as-you-go pension system leads to decreasing fertility, increasing high steady state equilibrium income and a possibility to escape a poverty trap. The introduction of an informal pension system to a pay-as-you-go pension system economy allows countries which are facing a sharp decrease in fertility to weaken this effect but reduces the long-run equilibrium per capita income.

Our analysis further highlights that the income level needed to escape a Malthusian stage of an economy is lower for a pay-as-you-go pension system. This allows us to follow that pension systems seem to play a vital role in the timing of demographic transition. The divergence of pension systems for developed and developing countries can therefore partly explain the observed regional differences in the behavior of population dynamics.

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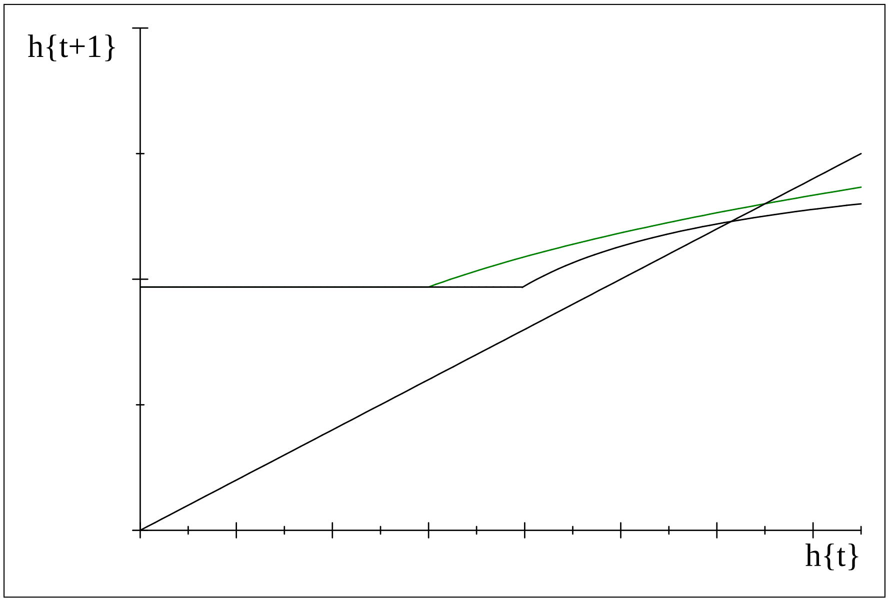
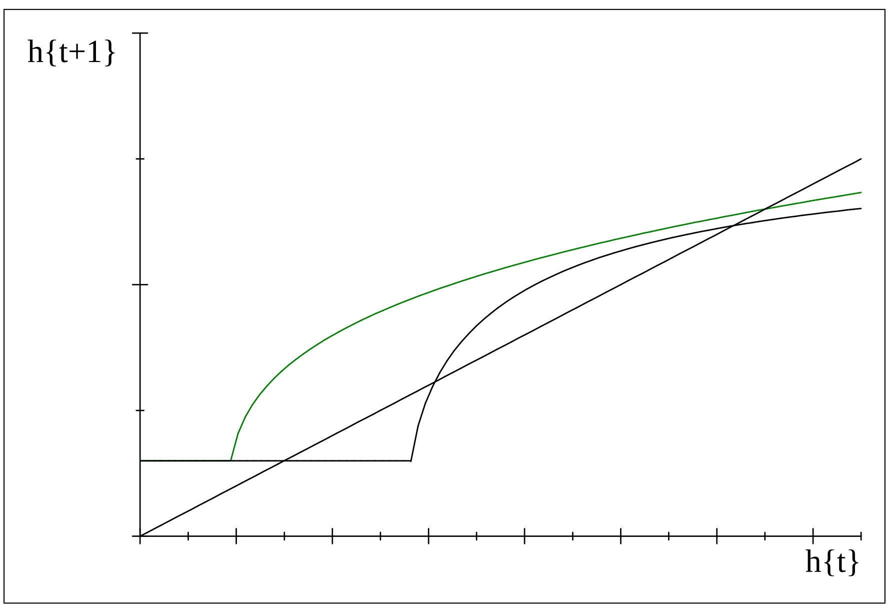
Appendix A

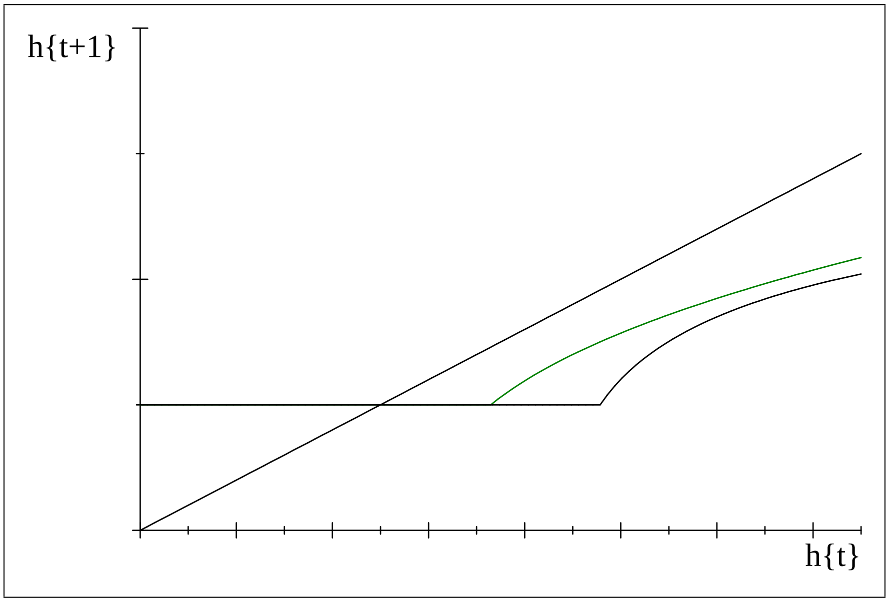
If $\eta a^{\sigma} > \overline{h}^{I}$:



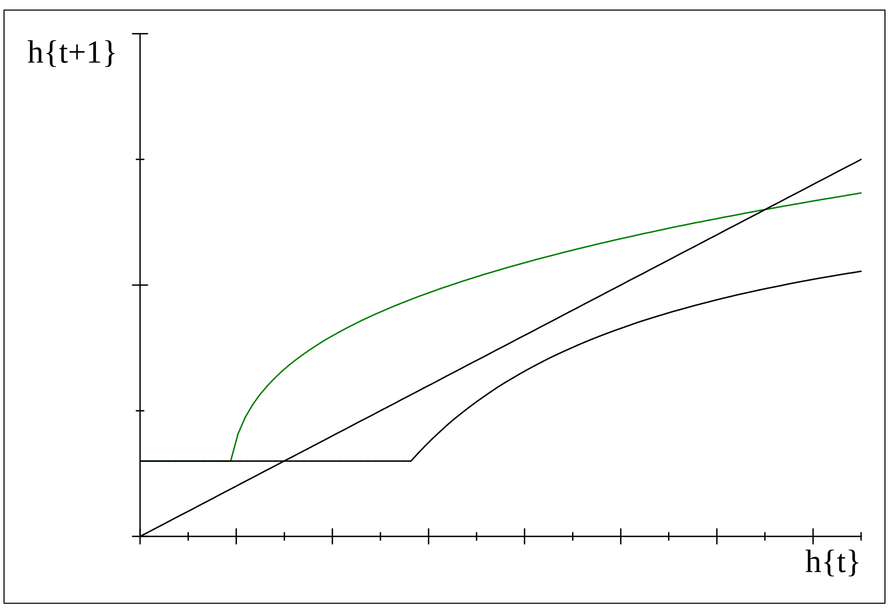
If $\overline{h}^{P} < \eta a^{\sigma} < \overline{h}^{I}; L(h^{'}) > R(h^{'})$:



$\eta a^{\sigma} < \overline{h}^{P}; L(h^{'}) < R(h^{'}):$



$\overline{h}^{P} < \eta a^{\sigma} < \overline{h}^{I}; L(h^{'}) < R(h^{'}):$



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