

Banking panics and deflation in dynamic general equilibrium*

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Abstract

Typically banking panics have been associated with deflation and declines in economic activity in the monetary history of the US and other countries. This paper develops a dynamic framework to study the interaction between banking and monetary policy. One result is the presence of multiple equilibria: banking panics and deflation arise at the same time and endogenously as equilibrium outcomes. Deposit contracts are written in nominal terms, so if prices fall relative to what was anticipated at the time the deposit contract was signed, then the real value of banks' existing obligations increases. So banks default, a banking panic precipitates and economic activity declines. If banks default on their deposits the demand for cash in the economy increases, because financial intermediation provided by banks disappears. The price level drops thereby leading banks to default. Friedman and Schwartz hypothesized that if the monetary authority had followed an alternative monetary policy during the early 1930s, aimed at keeping prices constant, banks would have been prevented from failing and output from falling, thus reducing the extent of the cycle. In the context of this model the Friedman-Schwartz hypothesis is correct. In this framework a mechanism like deposit insurance, when coupled with strict regulatory arrangements, achieves the same goal as the monetary policy. Absent strict regulatory arrangements however, deposit insurance amplifies business cycle fluctuations by inducing moral hazard.

JEL: E53, E58, G21, N12. Keywords: banking panics, deposit insurance.

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1 Introduction

This paper is motivated by the empirical observation that banking panics have been associated with deflation and declines in economic activity in the monetary history of the US and other countries. The main contribution of the paper is that it develops a model in which deflation and banking panics are mutually reinforcing and induce a decline in economic activity, therefore providing an explanation for such empirical observation. The paper also contributes to the literature on banking and monetary policy by evaluating the Friedman and Schwartz hypothesis that a more active monetary policy during the early 1930s would have prevented the collapse of the banking system and would have resulted in a milder cycle. The paper also evaluates the effectiveness of deposit insurance in preventing panics and its effects on aggregate output. Findings are that an active monetary policy aimed at keeping prices constant successfully prevents deflation and banking panics. Deposit insurance can prevent banking panics but generates larger output fluctuations than the monetary policy does because it induces moral hazard.

Evidence from Sprague [22] shows that the banking panics of 1873, 1884 and 1907 in the United States were all accompanied by a fall in the price level: typically the decline in prices of agricultural goods was more relevant than others, because of the extent to which it affected the value of banks' assets. Friedman and Schwartz [18] report that between 1865 and 1879 wholesale prices fell continuously at a rate approaching 6.5% a year, with a sharper decline between 1873 and 1879 of over 30% (p.30,32,42) when a banking panic occurred; between 1882 and 1885 they fell by

over 20% with the panic starting in 1884 (p.94); between 1892 and 1894 they fell by roughly 15% with bank runs precipitating in 1893 (p.94,108); the banking panic of October 1907 was associated with a fall in prices that reached a monthly rate of 5% (p.156).

The decline in real activity was also substantial: during the panic of 1873, loans by national banks fell on average by 9% and during the panic of 1907 they fell on average by 2% (Sprague, 1910, p.305-310). For the banking panics that occurred during the Great Depression Friedman and Schwartz [18] offer a detailed description of the extent of the fall in prices and economic activity: prices fell by 36% and industrial production by roughly 50% over the course of 1929-1933 (p.303). During the same time frame banking panics were frequent: the first panic occurred in October 1930 and deposits kept falling until January 1931, the second panic lasted from March 1931 through August 1931 and the final wave of panics precipitated in January 1933 ending with the Banking Holiday in March 1933 (Friedman and Schwartz, [18], p.308-328).

Looking at more recent episodes, during Japan's Lost Decade prices fell considerably (by 1.5% every year since mid 1990s until 2002) and real activity grew on average by only 1% every year during the period 1991-2002 (Baba et al., [1]): the existence of a deposit insurance agency prevented bank runs but banking difficulties and widespread banking failures were well known. Therefore the evidence for banking crisis and deflation occurring together is not only limited to the US: however a similar economic mechanism may have been at work in different economies at different points in time.

Research on banking panics has been very active in the last twenty years, and the available literature is very substantial. Numerous authors have argued that banking panics arise as multiple equilibria phenomena and lead to a decline in economic activity. Mostly they build on the Diamond and Dybvig [15] framework where banking panics are the result of a coordination failure among depositors, when they fear that the bank may be insolvent because other depositors suddenly withdraw their funds. The standard friction in these models is that banks' balance sheets have a maturity mismatch: they have long term assets but short term liabilities because of the nature of the deposit contract that entitles the depositor to claim payments on demand. This paper focuses on a different type of mismatch in banks' balance sheets that induces them to fail: banks have real assets but liabilities fixed in nominal terms¹. We may enrich the model by introducing the maturity mismatch as in the Diamond and Dybvig models, but for the purpose of this paper this is not relevant. In fact the economic mechanism we study abstracts from it.

This paper provides a model where banking panics and deflation arising endogenously due to self fulfilling expectations and reinforce each other: also, they are associated with declines in economic activity. When banking panics occur the public changes the composition of their portfolios switching from deposits to cash. With deposits falling also banks investment into productive projects decreases and therefore

¹We could think of banks making nominal loans to firms and argue that banks' assets are indeed nominal. However when firms have a nominal loan and real assets, that are the output of their productive projects, then firms themselves face the mismatch between real assets and nominal liabilities. If firms were to fail though, then the bank would seize the firm's output as a collateral: therefore banks' assets would be in real terms. Since we will be focusing on episodes of firms' and banks' failures then we can just think of banks' assets being in real terms.

economic activity is lower than it is when there are no panics.

The economic mechanism that induces banking panics when prices fall works through a mismatch in banks balance sheets between the value of banks assets and the value of banks liabilities. Banks can only offer nominal contracts to depositors. Therefore deposits, that are liabilities to banks, are at book value in their balance sheets: they are indexed to the price level of the time when the liability originated. Banks assets, on the other hand, are at market value because they are productive projects that banks invested in: they are indexed to the current price level. If prices unexpectedly fall then the real value of existing nominal obligations increases, whereas the real value of assets is unchanged, leading banks to be insolvent. Banks fail and depositors drastically reduce deposit holdings and increase cash holdings in their portfolios: hence a banking panic occurs.

The economic mechanism that drives deflation when there is a banking panic works through a decrease in the financial intermediation provided by banks when they fail. In the model banks play two roles: they finance productive projects and they issue liabilities that can be used as a means of payment. When there is a banking panic and banks fail, the liabilities that they issued are no longer a viable means of payment: financial intermediation provided by banks disappears and the only means of payment available to households in order to complete transactions, is cash.

Therefore during a banking panic households demand for cash increases and as a consequence prices fall. Economic activity then falls because banks stop investing in

productive projects since they have no longer funds available in the form of deposits when households shift away from deposits and demand cash. Therefore this model captures the main aspects of the banking panics that occurred before the onset of Federal Deposit Insurance in 1934: they took place in an environment where prices were falling, aggregate demand for liquidity increased and production fell.

Friedman and Schwartz [18] describe the contraction of the early '30s as a testimonial to the importance of monetary forces and to the role of monetary policy as a potent instrument for promoting economic stability. Their argument is based on the observation that the increased demand for liquidity in the economy was not matched by an increase in the stock of money. Without any increase in the stock of money, prices started to fall and banks were forced to liquidate their assets to face the public's demand for currency, which further reduced the value of their portfolios and forced them into insolvency. Friedman and Schwartz hypothesize that had the Federal Reserve System adopted an expansionary monetary policy, prices would not have dropped by over one-third in the course of four years and banking difficulties would have been appreciably eased. As a result the economic contraction might have been far less severe. In the environment described in this paper a version of the Friedman and Schwartz hypothesis is true: a monetary policy aiming at keeping prices constant would prevent deflation and, by easing banks difficulties, would also prevent banking panics and result in a milder cycle.

Many researchers ² believe that the introduction of deposit insurance at a federal

²Among many, Friedman and Schwartz [18], Calomiris and White [9], and White [23]

level in the U.S. was the result of the failure of monetary policy in avoiding the collapse of the banking system during the early 1930s. Among these authors, Friedman and Schwartz emphasize that the introduction of a federal deposit insurance scheme greatly reduced the need to rely on a response from the monetary authority to a change in the ratio of deposits to currency in households' portfolios, so that a banking panic, once begun, would not be permitted to cumulate.

This paper also evaluates the effectiveness of deposit insurance in preventing panics. Although deposit insurance can prevent banking panics, insuring banks deposits induces moral hazard on the side of banks. Having their liabilities always bailed out in a bad state of the world, banks choose to invest in more volatile projects that pay a higher return in the good state of the world. Therefore an economy with deposit insurance features larger aggregate output volatility than an economy with a monetary authority adopting an active monetary policy as suggested by Friedman and Schwartz. In this sense I argue that deposit insurance induces larger business cycle fluctuations.

2 Related literature

This paper is largely related to the literature that investigates the self-fulfilling feature of banking panics, started with Diamond and Dybvig [15]. In particular, Chari [11] studies a version of Diamond and Dybvig's model with bank specific risk and shows that there is a mechanism that can eliminate banking panics, conditional on the availability of a reserve technology and the existence of an interbank lending market.

Chari and Jagannathan [13] provide an information theoretic rationale for bank runs, building on Diamond and Dybvig's framework: banking panics in their environment occur because of a coordination failure among depositors who are uninformed about the state of the world affecting banks' assets' productivity, and observe a fraction of depositors withdrawing their deposits from the bank. Fearing that such withdrawals are based on information about the state of the world, uninformed depositors run on the bank. Ennis and Keister [16] argue that in a Diamond and Dybvig framework suspension of convertibility is not a time consistent mechanism for banks: they show that waves of bank runs may occur as equilibrium outcomes of a game where a bank that promised to suspend convertibility of deposits, if facing a run finds it optimal not to suspend payments. Green and Lin [19] show that in a Diamond and Dybvig framework where banks' sequential service constraint is formalized, the truth-telling equilibrium implements the symmetric, ex ante efficient allocation, therefore no runs occur: this optimal arrangement however is very different from the deposit contract we see in reality.

This line of research has raised several issues about whether the actual causes of a banks run coincide with those identified by the Diamond and Dybvig framework, and therefore whether this is the right framework to study bank runs and policies that could stop them or prevent them from happening. This paper provides a different framework and a different rationale for banking failures and panics. Households do not deposit when they know that banks are going to default, but banks default simply because the value of their assets unexpectedly decreases relative to the value of their

liabilities. There is no coordination failure among depositors here, so banking failures and a decline in aggregate deposits do not occur as a pure panic. The modeling strategy is to generate a panic-default equilibrium through a coordination failure on the side of banks: conditional of the realization of a sunspot banks expect a fall in prices that would drive them into insolvency, therefore they default and when they do so prices fall. The nature of the coordination failure is not crucial to the economic mechanism though: instead of conditioning banks' strategies on the realization of a sunspot we could think of an aggregate shock affecting the productivity of banks' assets so that when a bad shock hits banks' assets are not sufficient to cover banks' liabilities so that banks fail. As a modeling strategy we chose to work with a sunspot that has no real effect in order to avoid confusion about which specific economic mechanism is driving banking failures and deflation. The contribution of this paper is to study an amplification mechanism of a banking crisis through the feedback from a decline in financial intermediation caused by banking failures, onto prices and then from a fall in prices onto banks' balance sheets: therefore we preferred to keep this mechanism as simple as possible without any interactions with other possible causes for failures.

Also, a common feature of the papers in this literature is that they focus on banking panics arising because of a maturity mismatch in banks' balance sheets. As noted earlier, the environment developed in this paper abstracts from it³, rather it introduces a different type of mismatch in banks' balance sheets that is relevant

³Although it could be extended to include it.

for the economic mechanism underpinning the model: assets are in real terms and liabilities fixed in nominal terms. In this respect then this paper is related to earlier studies on the the interaction between deflation and bankruptcies, many of which focused on events occurred during the Great Depression. Fisher [17] argues that during the Great Depression the fall in prices was responsible for the massive number of bankruptcies: businesses and banks had nominal debt whose value increased in real terms as prices kept falling over the course of the Depression. He also argues that it was possible "to stop or prevent such a depression simply by reflating the price level up to the average level at which outstanding debts were contracted by existing debtors and assumed by existing creditors, and then maintaining that level unchanged" (p.346).

Friedman and Schwartz [18] argued that the banking and liquidity crisis played a crucial role during the Great Depression mainly because the crisis induced a decline in the stock of money that caused prices to fall⁴. The fall in prices induced by the initial banking failures, then caused the failure of many more banks, and the banking system eventually collapsed. Therefore Friedman and Schwartz argue that had the Federal Reserve System adopted an active monetary policy, since early on during the Depression, the decline in the stock of money relative to the increased demand for cash would have been halted and the resulting cycle would have been much milder.

This paper formalizes both Fisher's mechanism through which deflation causes banks and firms to fail, and Friedman and Schwartz's idea that banking panics and

⁴Friedman and Schwartz argue that the stock of money declined because deposits drastically decreased as the public demand for cash increased: absent any policy intervention then prices fell.

the resulting banking collapse induced a decline in the stock of money relative to the demand of cash in the economy and therefore lead to a fall in prices.

3 Model

The model economy is a dynamic game with a continuum in the interval $[0, 1]$ of identical households who are anonymous and a continuum in the interval $[0, 1]$ of identical banks. Banks are not anonymous: the history of their past actions is publicly observable. Time is discrete and infinite.

3.1 Households

Households are modeled similarly to the Lucas-Stokey (1987) cash credit economy: their preferences are defined over two types of goods, cash goods (c_1) and credit goods (c_2) and are represented by a utility function $U : R_+^2 \rightarrow R_+$, such that $U_i > 0, i = 1, 2$ and $U_{ii} < 0, i = 1, 2$. In every period households receive an endowment y and they have access to a technology that allows them to transform one unit of endowment into one unit of either cash goods or credit goods. Households are also endowed with a non perishable good, namely money, that they can use to transfer wealth intertemporally. They can also transfer wealth from one period to another by depositing into banks.

Each household is divided into a worker and a shopper: at the beginning of every period the asset market opens and the worker and the shopper make their portfolio decisions together, as a household. So they decide how much money to carry into

the period and how much to deposit at each bank. Then the goods' market opens and the worker and the shopper are separated from each other: the shopper takes the cash in his portfolio and purchases consumption goods at other households' location. The shopper is constrained to purchase cash goods paying right away using cash, whereas he can purchase credit goods for current consumption paying for them upfront using liabilities issued by banks, such as checks, debit cards and credit cards, that banks issued to households at the time deposits were made. So households have no obligations at all, while banks are responsible to settle those payments. Banks settle such payments at the end of the period by operating a standard clearing system. Notice that in this economy, when the banking system is operating, there is always inside money in the same amount as the value of the credit goods purchased. Households pay for credit goods using banks' issued liabilities that are then cleared at the end of the period among banks.

At the same time as the shopper purchases consumption goods the worker stays at home and produces cash or credit goods using the endowment y .

At the end of the period the shopper returns home and consumption takes place. Unspent cash is brought into tomorrow together with the gross return on the deposits made in the previous period and the income from the sales of the endowment.

3.2 Banks

The financial and productive sectors in this economy are consolidated and represented by banks. Therefore we should think of banks as if they were bankers and

entrepreneurs at the same time: bankers are intermediaries between depositors who supply their savings and entrepreneurs who demand funds to undertake productive projects. As a modeling strategy, we will let banks (i.e. bankers/entrepreneurs) have access to productive projects and carry them out. Banks' preferences are defined over credit goods⁵; let c_t^b denote banks' consumption at time t and γ denote banks' discount factor.

They have a fixed endowment L of labor in every period, and they have access to a productive technology $f : R_+^2 \rightarrow R_+, f_i > 0, i = 1, 2$. The inputs to the productive technology are an investment of cash good⁶ and the fixed factor L .

Banks behave competitively. They offer deposit contracts to households and carry out production: the type of contract they can offer is such that the rate of return on deposits is fixed in nominal terms⁷. The deposit contract between households and banks allows households to hold a diversified portfolio of deposits⁸.

Besides being intermediaries between lenders and borrowers, in this economy banks also play a role in the payment system: they issue liabilities that can be used as a means of payment, up to the face value of the gross return on deposits and the income that households will receive at the end of the period for the sales of the

⁵This assumption is not crucial: banks' preferences may be defined on cash goods instead. Alternatively cash and credit goods may be treated as perfect substitutes as far as banks' preferences are concerned.

⁶This assumption is not crucial: none of the results would change if the input to the productive technology was a credit good. By assuming it is a cash good we are generating a demand for cash on the side of banks: banks need cash in order to purchase production inputs so they want to sell deposits to households in order to raise that cash.

⁷This restriction is meant to capture one of the key features of deposit contracts in reality.

⁸Results are unchanged however if the deposit contract is one-to-one: each household can choose which bank it wants to deposit at, but only one bank. Also, each bank can take the deposits only of a single household.

endowment. Therefore households who decided to deposit part of their assets in a bank, are able to make payments up to the nominal value of the gross return on their deposits and the income they will receive at the end of the period for the sales of the endowment using banks' issued liabilities. The role of banks as providers of financial intermediation is crucial for the results.

3.3 Timing of players' moves

At the beginning of every period the outcome of a random variable (sunspot) θ_t is publicly observed. θ_t has range $\{0, 1\}$ and distribution Π :

$$\begin{cases} \Pi(\theta_t = 0) &= \pi \\ \Pi(\theta_t = 1) &= (1 - \pi) \end{cases}$$

We will say that the sunspot hits at time t if $\theta_t = 1$.

At time t , after the realization of the sunspot, θ_t , banks simultaneously choose whether to default or not: if they don't default they sell the output from the productive technology on the goods' market, they pay households back for the deposits they made and the interest rate that it was promised to them as a return on deposits. If banks choose to default then depositors are not paid back, not only for the promised return but also for the actual deposits previously made. So households who deposited lose not only the return on their investment but also the assets they invested when banks default. In other words the gross interest rate on deposits when banks default is zero⁹. The default decision of bank j at time t is denoted $\delta_t(j)$.

⁹Allowing banks to keep their assets when they default on their liabilities is not crucial: however

After banks have decided whether to default or not, households choose their consumption allocation (c_{1t}, c_{2t}) and asset holdings (M_t, D_t) and banks decide how much to invest in the productive technology¹⁰ using the deposits they sold to households (D_t^b) . At the end of the period, if banks did not default, then depositors get the promised return on the deposits made in the previous period $(R_{t-1}D_{t-1})$.

The timing of players' moves is represented in Figure 1:

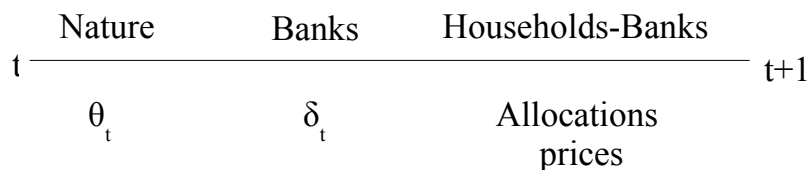


FIGURE 1

So for every period t a stage game can be defined, where Nature first draws a realization of θ_t , then banks simultaneously make their default decision. After having observed banks' default decision, households choose consumption allocation and asset holdings (in particular they choose whether to deposit or not and whether to withdraw their deposits from banks). Then banks move again and choose how

it is crucial that when banks default the payment system gets disrupted and households must use cash to pay for their consumption purchases. If we seized banks' assets when they default, for instance, and rebated them to households after a liquidation process of the banks takes place, the key results of the paper would not be affected as long as during the liquidation process households cannot use all of the payments they will receive from the liquidation of banks' assets: results would hold however for a different parameters' space.

¹⁰One factor of production is an investment of cash good, so banks need cash to be able to purchase it and carry out production.

much to invest in the productive technology, and at the very end of the period households are paid back for the deposits they made in the previous period if banks did not default on their deposits at the beginning of the period.

The stage game is represented in extensive form in Figure 2, where after Nature has drawn a realization of θ_t , bank j chooses whether to default or not without knowing what other banks j' chose, and then households choose consumption allocation and asset holdings and banks choose how much to invest in the productive technology. In particular households choose whether to deposit a strictly positive amount of assets or not. For analytical tractability it is assumed that when a bank defaults

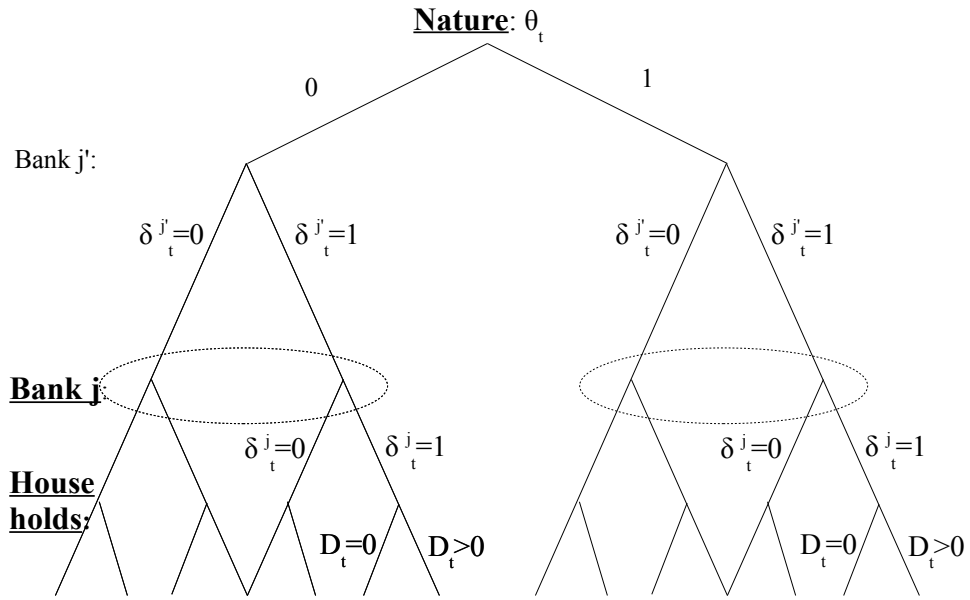


FIGURE 2

then it loses its endowment of labor L forever after. Therefore letting $\lambda_t = \int_0^1 \delta_t(j) dj$ denote the measure of defaulting banks at time t , if a measure one of banks defaults

at time t ($\lambda_t = 1$) then the banking system shuts down forever and the only source of output in the economy is households' endowment.

3.4 Players' actions and strategies

Let the relevant history of the game at the beginning of time t be anonymous with respect to households, since they are anonymous players in the game, and be denoted:

$$h^{t-1} = (\delta_s(j)_{j \in [0,1]}, \theta_s, p_s, R_s, A_s, c_{1s}, c_{2s}, M_s, D_s(j)_{j \in [0,1]} \mid s \leq t-1)$$

that is a list of all the past default decisions by every bank $j \in [0, 1]$ ($\delta_s(j)_{j \in [0,1]}$), sunspot realizations, prices of consumption goods (p_s) and deposits (R_s), aggregate households' assets at the beginning of every period (A_s), aggregate consumption of cash good (c_{1s}), aggregate consumption of credit good (c_{2s}), aggregate cash (M_s) and deposits holdings at every bank j ($D_s(j)$).

Let the history of the game at time t after banks' default decisions have been made be denoted:

$$h_1^t = (h^{t-1}, \theta_t, \delta_t(j)_{j \in [0,1]})$$

that includes the history at the beginning of period t , the current realization of the sunspot (θ_t) and the current default decision of every bank j ($\delta_t(j)_{j \in [0,1]}$).

Let the set of possible histories at the beginning of time t be denoted H^t , with $H^0 = \emptyset$, and the set of possible histories at time t after banks' default decisions have been made be denoted H_1^t with $H_1^0 = \{\theta_0, \delta_0(j)_{j \in [0,1]}\}$, so that h_1^t is a typical element of H_1^t .

An action for a household is a choice of consumption of cash and credit goods, deposits and cash holdings, and assets to carry into the next period. A strategy is a mapping $\sigma_t^H : H_1^t \rightarrow \mathbb{R}_+^5$. When history h_1^t is realized, households' strategy is denoted:

$$\sigma_t^H(h_1^t) = \{(c_{1t}(h_1^t), c_{2t}(h_1^t), D_t(h_1^t), M_t(h_1^t), A_{t+1}(h_1^t)) \in \mathbb{R}_+^5\}$$

and a strategy profile for a representative household is denoted $\sigma^H = \{\sigma_t^H\}_{t=0}^\infty$.

Let $\mu_t^H : H_1^{t+1} \rightarrow [0, 1]$ denote the conditional probability¹¹ that history $h_1^{t+1} \succ h_1^t$ will be realized if h_1^t is the realized history at time t and recall that $\lambda_t = \int_0^1 \delta_t(j) dj$ denotes the measure of defaulting banks at time t . Then a household chooses $(c_{1t}, c_{2t}, M_t, D_t, A_{t+1})$ to solve:

$$v_t(h_1^t, A_t, D_{t-1}) = \max\{U(c_{1t}, c_{2t}) + \beta \sum_{h_1^{t+1}} \mu_t(h_1^{t+1} | h_1^t) v_{t+1}(h_1^{t+1}, A_{t+1}, D_t)\} \quad (1)$$

s.t.

$$M_t + D_t = A_t \quad (2)$$

$$p_t(H^t) c_{1t} \leq M_t \quad (3)$$

$$p_t(H^t) (c_{1t} + c_{2t}) \leq M_t + (1 - \lambda_t) (p_t(H^t) y_t + R_{t-1}(H^{t-1}) D_{t-1}) \quad (4)$$

$$\begin{aligned} A_{t+1} &= M_t - p_t(H^t) c_{1t} - p_t(H^t) c_{2t} \\ &\quad + p_t(H^t) y_t + (1 - \lambda_t) R_{t-1}(H^{t-1}) D_{t-1} \end{aligned} \quad (5)$$

¹¹induced by the distribution of Θ and players' strategies.

where constraint (2) is a securities market constraint: the household splits his assets between cash to carry within the period and deposits into banks. Constraint (3) is a cash in advance constraint on cash goods: the value of purchases of cash goods cannot exceed the value of cash brought within the period. Constraint (4) is a credit good constraint: credit goods can be purchased with unspent cash on cash goods, and a fraction, proportional to the measure of non defaulting banks, of the income from the sales of the endowment and the return on previous period deposits. Therefore if a measure one of banks defaults ($\lambda_t = 1$) then both cash goods and credit goods must be purchased using cash. On the other hand, if a measure zero of banks defaults ($\lambda_t = 0$) then households can pay upfront for credit goods' purchases using bank's issued liabilities up to the value of the income that they will receive at the end of the period for selling the endowment, and the return on previous period deposits. Banks provide financial intermediation by issuing liabilities that are accepted as means of payment: the upper bound on the value of liabilities that they issue is given by households' end of period income from the sales of the endowment and households' return on previous period deposits, which will both be paid at the end of the period. Exactly because both are paid at the end of the period, it would not be feasible to use them to pay for consumption purchases if banks were not providing intermediation in the form of liabilities that can be used to make payments. Constraint (5) is the law of motion for assets: assets at the beginning of the next period will be given by unspent cash, income from the sales of the endowment and a fraction, proportional to the measure of non defaulting banks, of the return on previous period deposits.

So if a measure one of banks defaults households lose a fraction of their wealth: the return on deposits made in the previous period. They also lose means of payment: they can no longer pay upfront for consumption purchases up to the value of their income using banks' issued liabilities. Households however don't lose their income as a form of wealth: they will still get paid for the sales of their endowment at the end of the period, and that is why their income enters the law of motion for assets.

An action for banks at the information set where they first move is the choice to default or not and a strategy is a mapping $\sigma_{1t}^B : H^{t-1} \rightarrow \{0, 1\}$. At the second information set where they move, an action for banks is a choice of investment into the productive technology and a choice of deposits to offer households. A strategy is a mapping $\sigma_{2t}^B : H_1^t \rightarrow \mathbb{R}_+^2$. When history h_1^t is realized, a strategy for bank j is denoted: $\sigma_t^B(j)(h_1^t) = (\sigma_t^B 1(j)(h^{t-1}), \sigma_{2t}^B(j)(h_1^t)) = \{d_t(j)(h^{t-1}), i_t(j)(h_1^t), D_t^b(j)(h_1^t)\}$ and a strategy profile is $\sigma^B = \{\sigma_t^B(j)_{j \in [0,1]}\}_{t=0}^\infty$.

Banks' first decision problem is to choose whether to default or not: if the real value of their liabilities exceeds the real value of their assets then they must default because they are illiquid. Banks do not have enough assets at time t to pay their time t obligations. Even if a bank were to sell all of its assets it would not be able to pay its liabilities: this is denoted as involuntary default state. If the real value of a bank's assets are sufficient to cover the real value of its liabilities then the bank chooses whether to default or not:

- if $f(i_{t-1}, L) < \frac{R_{t-1} D_{t-1}^b}{p_t} \Rightarrow$ involuntary default

- if $f(i_{t-1}, L) \geq \frac{R_{t-1}D_{t-1}^b}{p_t} \Rightarrow$ banks choose default or not

Notice that the involuntary default is related to banks being illiquid, rather than insolvent. Banks that default in this environment may well be solvent in the sense that the expected stream of future profits exceeds the current loss. Therefore if they were allowed to borrow inter-temporally against their future assets they might not need to default. However not allowing for such inter-temporal borrowing and lending is meant to capture those banking failures due to illiquidity only. If depositors have claims with banks, that mature at time t but that banks cannot meet at time t , then the bank has to fail.

Let $c_t^{b,j}(h^{t-1}, \theta_t, \delta_t(j))$ denote the payoff to bank j at time t after observing history h^{t-1} and sunspot realization θ_t , and making default decision $\delta_t(j)$. Then:

$$c_t^{b,j}(h^{t-1}, \theta_t, \delta_t(j)) = \begin{cases} f(i_{t-1}, L) - \frac{R_{t-1}D_{t-1}^b}{p_t} + \gamma w_{t+1}^j(h_1^t) & \text{if } \delta_t(j) = 0 \\ f(i_{t-1}, L) & \text{if } \delta_t(j) = 1 \end{cases}$$

with γ being banks' discount factor and with $w_{t+1}^j(h_1^t)$ being the value of bank j expected future profits at time t after history h_1^t . Then:

$$\text{if } \frac{R_{t-1}D_{t-1}^b}{p_t} \leq \gamma w_{t+1}^j(h_1^t) \quad \text{then bank } j \text{ does not default;} \quad (6)$$

$$\text{if } \frac{R_{t-1}D_{t-1}^b}{p_t} > \gamma w_{t+1}^j(h_1^t) \quad \text{then bank } j \text{ defaults.} \quad (7)$$

Banks' second decision problem is to choose how many deposits to sell and how much to invest in the productive technology in order to maximize expected profits:

$$w_{t+1}^j(h_1^t) = \max_{\{i_t, D_t^b\}} E_{\theta_{t+1}|\theta_t}[c_{t+1}^{b,j}(h^t, \theta_{t+1}, \delta_{t+1}(j)) | \sigma^B, \sigma^H] \quad (8)$$

$$s.t. \quad p_t i_t \leq D_t^b \quad (9)$$

Constraint (9) says that banks can finance investment into productive projects only up to the value of the deposits they sold. It reflects the implicit assumption throughout the paper that banks' discount factor γ is small enough¹² so that bankers consume their profits in every period. That allows us to bring into this framework the idea that firms¹³ are borrowing constrained when undertaking investment projects as pointed out in Bernanke and Gertler [4], Bernanke, Gertler and Gilchrist [5], and Carlstrom and Fuerst [10].

3.5 Equilibrium

The focus of the paper is on symmetric subgame perfect equilibria, therefore an equilibrium is defined as follows:

Definition 1 *A symmetric equilibrium is:*

1. a symmetric strategy profile for households $\sigma^H = \{\sigma_t^H\}_{t=0}^\infty$

¹²For a proof see Appendix, section *Banks impatient enough*.

¹³In this environment they are banks/firms.

2. a symmetric strategy profile for banks $\sigma^B = \{\sigma_t^B\}_{t=0}^\infty$

3. pricing functions $p_t(h_1^t), R_t(h_1^t)$ ¹⁴

such that for any t, h_1^t , households maximize; for any t, h^t , banks maximize and prices clear the markets:

$$c_t^b(h_1^t) + c_{1t}(h_1^t) + c_{2t}(h_1^t) + i_t(h_1^t) = y_t + f(i_{t-1}(h_1^{t-1}), L) \quad (10)$$

$$M_t(h_1^t) + D_t(h_1^t) = \bar{M}_t = \bar{M} \quad (11)$$

where constraint (10) is the resource constraint: $c_t^b(h_1^t)$ is aggregate consumption by banks when realized history is h_1^t , $c_{1t}(h_1^t)$ is aggregate consumption of cash good, $c_{2t}(h_1^t)$ is aggregate consumption of credit good, $i_t(h_1^t)$ is investment in the productive technology, $f(i_{t-1}(h_1^{t-1}), L)$ is the output of the technology that is realized at t using inputs from $t - 1$. Constraint (11) is the money market clearing condition where $M_t(h_1^t)$ stands for aggregate cash holdings by households within the period, $D_t(h_1^t)$ for the aggregate deposits holdings, \bar{M}_t for the stock of money supply at time t . We will focus on equilibria with constant money supply so that $\bar{M}_t = \bar{M}$.

¹⁴Notice that pricing functions are defined over aggregate histories: $\mathbb{H}^t = (h^{t-1}, \theta_t, \int_0^1 d_t(j) dj)$. However aggregate histories are functions of $h_1^t = (h^{t-1}, \theta_t, (d_t(j))_{j \in [0,1]})$, in that they are defined over the aggregate default decisions by banks rather than on each bank j default decision. Therefore ultimately pricing functions p_t and R_t are defined over histories h_1^t .

4 Equilibrium characterization

In this economy there are multiple equilibria. However we will focus on one equilibrium where banking failures and deflation are mutually reinforcing. A fall in prices drives banks to default and because banks default prices fall. In order to argue the existence of such an equilibrium and to characterize it, we assume:

Assumption 1 $U(c_1, c_2) = \log(c_1) + \log(c_2)$.

Assumption 2 $f(i_t, L) = i_t^\alpha L^{1-\alpha}$

Assumption 3 $\gamma < \beta^2 \pi$

Assumption 4 $[\alpha \beta^2 L^{1-\alpha} \pi]^{\frac{1}{1-\alpha}} < y$

Assumption 5 $\frac{\alpha(1-\gamma)\pi}{\pi\gamma(1-\alpha)} < 1$

Assumption 6 $\frac{y\alpha}{\pi\bar{r}} \left[1 + \frac{1}{2\beta(1-\pi) + \beta\pi \frac{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} \right] > y + y^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha$

Assumption 7 $\frac{\alpha\bar{r}}{\pi} < 1$

The following proposition shows that there exists a symmetric equilibrium such that no default occurs until the first date t at which the sunspot hits ($\theta_t = 1$), then banks fail and there is no banking system at any date after the sunspot hits¹⁵. In this

¹⁵We choose to characterize an equilibrium where we switch from no banking failures and high prices to banks failing, shutting down forever and low prices, for analytical tractability. Letting banks come back in business after failing would require evaluating banks' continuation payoffs at allocations that are not time invariant: although it is possible to characterize the properties of these allocations, we are unable to prove the existence of sequences of allocations and prices that solve the system of first order conditions and market clearing (i.e. an equilibrium) and are not time invariant (since we have already characterized the time invariant one).

equilibrium banking failures are associated with a fall in prices and with declines in economic activity.

Proposition 1 *Let t denote the first date at which the sunspot hits ($\theta_t = 1$). If Assumptions 1-7 are satisfied then there exists a symmetric equilibrium such that:*

- $\lambda_s = \int_0^1 \delta_s(j) dj = 1$, $D_s(j) = 0 \ \forall j \in [0, 1]$, $p_s = p^d$, $\forall s \geq t$
- $\lambda_s = \int_0^1 \delta_s(j) dj = 0$, $D_s(j) = D^{nd} > 0 \ \forall j \in [0, 1]$, $p_s = p^{nd}$, $\forall s < t$
- $p^d < p^{nd}$
- $$\begin{cases} Y_s = y & \forall s \geq t + 1 \\ Y_s = y + f(i(h_1^s), L) & \forall s \leq t \end{cases}$$

Proof.

See Appendix.

■

The intuition behind this result is the following: banks default when the actual price is too low relative to the price their nominal liabilities are indexed to. In fact if prices are low enough the real cost of paying depositors back exceeds the real value of banks' assets. When banks default, households can no longer use the liabilities issued by banks as a means of payment because once the payment is settled using banks' issued liabilities, banks themselves are responsible for those payments while households have no obligations at all. Therefore when banks fail nobody will accept their liabilities as payments: by defaulting banks can no longer be held responsible

for their obligations. As a consequence the set of goods that are purchased using cash expands because the transactions that used to be made with credit, now require a cash payment: households demand more cash in order to purchase consumption goods relative to the cash they would carry in an environment with no banking failures. Prices fall because the amount of consumption goods that are purchased using cash is large relative to the amount of cash brought within the period¹⁶. So that banking failures and deflation endogenously arise at the same time as equilibrium outcomes. In this equilibrium, when banks fail households do not deposit their assets. Therefore no investment into productive technologies takes place, since deposits are the only source of funds for banks to purchase production inputs. So aggregate output falls. Therefore this equilibrium features banking panics and deflation being mutually reinforcing and being associated with declines in economic activity.

5 The game with a Monetary authority

The goal of this section is to carry out a policy exercise in this environment in the same spirit as the Friedman and Schwartz hypothesis: if the monetary authority commits to a policy of keeping prices constant then banking panics and deflation are no longer an equilibrium outcome. So even when there is a sunspot prices do not fall, the banking system does not collapse and economic activity does not decline. We will focus on a one time policy experiment.

Let the game be modified so that there is another player, the Monetary Authority

¹⁶which occurs in equilibrium if $\pi \in (\underline{\pi}, \bar{\pi})$.

who moves after banks have decided whether to default or not. So the relevant history of the game for the Monetary Authority is h_1^t .

Let the economy start at time 0 with initial deposits D^{nd} and let the Monetary Authority adopt the following policy: if a positive measure of banks defaults at time 0 then it injects cash on the securities market to households in the amount $T_0(h_1^0)$, otherwise it leaves \bar{M} unchanged. The amount of the money injection $T_0(h_1^0)$ is just enough to keep current prices constant with respect to the previous period, and it depends on the measure of banks that defaulted at the beginning of the period. If a measure one of banks defaults ($\lambda_0 = 1$) then $T_0(h_1^0) = p^{nd}y - \bar{M}$ so that the new stock of money supply is $\bar{M}'(h_1^0) = \bar{M} + T_0(h_1^0) = p^{nd}y$. Then time 0 prices in equilibrium are $p_0 = p^{nd}$ because when the banking system collapses ($\lambda_0 = 1$) any consumption good that households buy must be paid in cash (their cash in advance constraint is $p_0(c_{10} + c_{20}) = \bar{M}'(h_1^0)$), and they buy a total of y goods. If a smaller measure of banks defaults then the cash transfer necessary to keep prices at no-default level (p^{nd}) will be smaller as well¹⁷. Also, since we are focusing on a one time policy experiment, we assume that if the Monetary Authority adopts an active policy at time 0 then it is not feasible for it to adopt an active policy $\forall t > 0$. Then with an active monetary policy, when the sunspot hits it is no longer optimal for banks to default, and the unique pure strategy equilibrium of the game is no default and no panics.

¹⁷In fact if the measure of defaulting banks is positive but strictly smaller than one ($\lambda_0 \in (0, 1)$) then equilibrium prices fall with respect to p^{nd} but not as much as they would with the banking system collapsing ($\lambda_0 = 1$) when equilibrium prices are p^d . Therefore the cash injection necessary to bring prices back up to p^{nd} would also be smaller.

Proposition 2 *If Assumptions 1-7 are satisfied then with an active monetary policy no default and no panics at time 0 is the unique pure strategy equilibrium and economic activity does not decline.*

Proof.

See appendix.

■

The result of Proposition 2 is a validation of the Friedman-Schwartz hypothesis: had the Federal Reserve adopted an active monetary policy during the early 1930s, prices would not have fallen, banking panics would not have arisen to the extent of inducing a collapse in the banking system, and the resulting cycle would have been much milder.

Monetary policy in this environment is very powerful: banking crises arise solely from a fall in prices because banks' liabilities are fixed in nominal terms. By controlling the stock of money in the economy, the Monetary Authority is able to influence prices, and in this framework knowing that the Monetary Authority is adopting an active policy is enough to give banks incentives not to default. Therefore if there is public information about monetary policy aiming at keeping prices constant, then no banks will fail and no deflation will occur. Depositors will not panic and economic activity will not decline. Notice that on equilibrium path no actual cash injection takes place. Adopting an active monetary policy is sufficient to discipline off equilibrium payoffs so that certain strategies can be implemented as part of an equilibrium.

6 The game with Deposit Insurance

The goal of this section is to show that in this environment a mechanism like deposit insurance, when coupled with strict regulatory arrangements, is able to prevent banking panics and deflation. Absent strict regulatory arrangements however, deposit insurance can prevent banking panics and deflation but it induces larger output fluctuations than the monetary policy analyzed in the previous section because it induces moral hazard on the side of banks.

Since we are interested in introducing the concept of moral hazard, we enlarge the set of productive technologies that banks can invest in to include both a safe and a risky technology:

- safe technology

$$f(i_t, L) = i_t^\alpha L^{1-\alpha} \tag{12}$$

- risky technology

$$\hat{f}(i_t, L) = \hat{r}_t^\alpha L^{1-\alpha} \tag{13}$$

with \hat{r} being an aggregate shock with range $\{\underline{r}, \bar{r}\}$ and probability distribution:

$$\begin{cases} P[\hat{r} = \bar{r}] &= q \\ P[\hat{r} = \underline{r}] &= (1 - q) \end{cases}$$

and such that $q\bar{r} + (1 - q)\underline{r} < 1$ so that technology \hat{f} is a mean reducing spread of

technology f . The distribution of the random variable \hat{r} is assumed to be independent of the distribution of the sunspot θ and is *i.i.d* over time. Without loss of generality we normalize $\underline{r} = 0$.

As in the previous section let the economy start at time 0 with initial deposits D^{nd} and let households and banks design a mechanism like deposit insurance: a deposit insurer is set up, that seizes banks' assets if they default and pays depositors the amount they were promised every time banks fail. Deposit insurance is set up after households' deposit decision but before banks' choice of technology. Since households deposit before banks' choice of technology we assume that the payment schedule that banks offer in the deposit contract is associated with investment in the safe technology, but allow banks to renegotiate the contract afterwards¹⁸. Similarly to the monetary policy experiment we focus on a one time deposit insurance, following which the mechanism is no longer feasible to establish.

In this environment deposit insurance is a mechanism such that when banks fail and depositors are bailed out, the deposit insurer seizes banks' assets and takes on banks' liabilities, therefore making it still possible to use the liabilities issued by failed banks as a means of payment. In other words, the deposit insurer guarantees that those liabilities are going to be payed back. Also any resources left over after paying depositors when banks fail, are lump sum transferred back to banks. On the other hand, if seizing banks' assets is not enough to cover payments to depositors, then

¹⁸This means that banks offer interest rate payments consistent with their expected profits maximization under investing in the safe technology. If they choose the risky technology when they actually make their investment decision, then they can renegotiate the payment schedule in the contract if they want to.

the deposit insurer levies a lump sum tax on households' endowment to finance the remaining payments to depositors.

Further it is assumed that households can observe the technology that banks invest in after they made their deposit decision but before deciding whether to withdraw.

Assumption 8 $q > \underline{q} = \frac{[1 - \frac{\alpha}{\pi} + \frac{\gamma(1-\alpha)}{(1-\gamma\pi)}]}{[\bar{r} - \frac{\alpha}{\pi} + \frac{\gamma(1-\alpha)}{(1-\gamma\pi)}]}$

Proposition 3 *If Assumptions 1-8 then:*

1. *without deposit insurance it is a strictly dominant strategy for banks to invest in the safe technology f ,*
2. *with deposit insurance and with strict regulations (banks have only access to f) then no default and no panics at time 0 is the unique pure strategy equilibrium*
3. *with deposit insurance but without strict regulations then the unique pure strategy equilibrium at time 0 is such that:*
 - *banks invest in \hat{f} ,*
 - *banks do not default and households do not panic if $\hat{r} = \bar{r}$,*
 - *banks default but households are paid if $\hat{r} = \bar{r}$.*

Proof.

See appendix.

■

This result is twofold: on one hand it validates the effectiveness of deposit insurance in preventing banking panics, as it has been the case in U.S banking history since the establishment of the Federal Deposit Insurance Corporation in 1934. On the other hand it highlights the effects that the nature of such an insurance contract has on output. Because the deposit insurer takes on banks' liabilities in a bad state of the world, without affecting bank's profits in a good state of the world, it creates moral hazard on the side of banks. Banks have incentives to invest in more risky assets because they have a higher payoff in case of success of the project they invest in, but do not have to bear any loss in case of failure. In fact, if the deposit insurer cannot force banks to invest in the safe technology then banks choose to invest in the risky one. Therefore, while preventing banking panics, deposit insurance generates larger output volatility than a monetary policy à la Friedman and Schwartz because it induces moral hazard. Because banks invest in the risky technology, in good states of the world aggregate output is high and in bad states of the world it is low: in this sense I argue that deposit insurance amplifies business cycle fluctuations. In fact if the deposit insurer could force banks to invest only in the safe technology, then deposit insurance would achieve the same outcome as the one induced by a monetary policy that aims at keeping prices constant, as proven in Proposition 3 part 2. Also, Proposition 3 part 1. guarantees that none of the previous result from Propositions 1 and 2 changes when banks are allowed to choose between a safe and a risky technology: without deposit insurance banks choose to invest in the safe technology f , and therefore Propositions 1 and 2 hold.

7 Conclusion

This paper is motivated by the empirical observation that banking panics have typically been associated with deflation and declines in economic activity in the monetary history of the U.S. and other countries. Therefore its main contribution is that it develops a framework where these events all arise endogenously as equilibrium outcomes. None of them is taken as exogenously given, they are all the result of self-fulfilling expectations.

Two economic mechanisms are crucial for this result. The first one works through deposit contracts between households and banks being fixed in nominal terms. This makes an unanticipated fall in the price level affect the real value of existing obligations to the extent that it exceeds the real value of banks' assets, thus forcing banks to default on their liabilities. The second one works through a change in the financial intermediation provided by banks: when banks are in business they issue liabilities that are used as a means of payment, so that certain transactions in the economy are carried out without the need to pay with cash. When banks fail however such liabilities are no longer backed up by banks, so they are no longer a viable means of payment and any transaction in the economy has to be carried out using cash. The set of goods that are purchased using cash expands during banking failures, so that households demand for cash increases and prices fall.

Therefore the innovation of this paper is to bring together features of models with nominal debt and models with cash and credit goods and to show that when these key features coexist and are linked by the banking system, then banking panics

and deflation are mutually reinforcing and banking crisis may be amplified through deflation.

The paper also carries out two different policy experiments: the first concerns the effectiveness of an active monetary policy similar to the one suggested by Friedman and Schwartz for the early 1930s, in preventing banking panics, deflation and decline in economic activity. The second concerns the effectiveness of a deposit insurance mechanism in attaining the same result. Findings are that both the monetary policy and deposit insurance can prevent banking panics, deposit insurance however, by inducing moral hazard, generates larger output fluctuations.

A Appendix

A.1 Proof of Proposition 1

Proof. By construction. Let:

- $\delta_t(j) = 0$ if $\theta_t = 0$ and $D_t(j) > 0$ if $\delta_t(j) = 0$
- $\delta_t(j) = 1$ if $\theta_t = 1$ and $D_t(j) = 0$ if $\delta_t(j) = 1$

A strategy for bank j is constructed so that if the sunspot hits then bank j defaults, and bank j doesn't default otherwise. A strategy for a household i is constructed so that if bank j defaults then no households $i \in [0, 1]$ will deposit in bank j . Then before the sunspot hits we look for a time invariant consumption allocation (c_1^{nd}, c_2^{nd}) and asset holdings (M^{nd}, D^{nd}) that solve the households' problem, investment into

the productive technology and deposits offered by banks $(i^{nd}, D^{b,nd})$ that maximize banks' expected profits, and clear the markets at prices p^{nd}, R^{nd} , where the superscript nd stands for no default given the constructed strategy profiles. Optimality conditions for the households' problem are:

$$p_t c_{1t}^{nd} = M_t^{nd} \quad (14)$$

$$p_t(c_{1t}^d + c_{2t}^d) = M_t^d = \bar{M}. \quad (15)$$

$$\frac{U_{1t}^{nd}}{p_t^{nd}} \geq \beta R_t \pi \frac{U_{2t+1}^{nd}}{p_{t+1}^{nd}} \quad \text{with " = " if } D_t > 0 \quad (16)$$

$$\frac{U_{2t}^{nd}}{p_t^{nd}} = \beta \left((1 - \pi) \frac{U_{1t+1}^d}{p_{t+1}^d} + \pi \frac{U_{1t+1}^{nd}}{p_{t+1}^{nd}} \right) \quad (17)$$

$$\frac{U_{1t}^{nd}}{p_t^{nd}} = \beta \pi R_t \left(\beta (1 - \pi) \frac{U_{1t+2}^d}{p_{t+2}^d} + \beta \pi \frac{U_{1t+2}^{nd}}{p_{t+2}^{nd}} \right). \quad (18)$$

where superscripts nd, d stand for no default and default, which is equivalent to indicating whether the realization of the sunspot hits ($\theta_t = 1$) or not ($\theta_t = 0$), given the constructed strategy profiles.

From banks profit maximizing condition we also have that¹⁹: $R_t = \frac{p_{t+1}^{nd}}{p_t^{nd}} \frac{1}{\pi} f_1\left(\frac{D_t}{p_t^{nd}}, L\right)$

¹⁹Banks' choice of deposit to sell and investment into the productive technology taking as given the constructed strategy solves:

$$\begin{aligned} \max_{i_t, D_t^b} & \quad \left[\pi \left(f(i_t, L) - \frac{R_t D_t^b}{p_{t+1}^{nd}} \right) + (1 - \pi) f(i_t, L) \right] \\ \text{s.t.} & \quad p_t i_t \leq D_t^b \end{aligned}$$

whose first order necessary conditions are: $R_t = \frac{p_{t+1}^{nd}}{p_t^{nd}} \frac{1}{\pi} f_1\left(\frac{D_t}{p_t^{nd}}, L\right)$ and $p_t^{nd} i_t = D_t^b$

which with a Cobb Douglas technology as specified in assumption (??) is:

$$R_t = \frac{p_{t+1}^{nd}}{p_t^{nd}} \frac{\alpha(i^{nd})^\alpha L^{1-\alpha}}{\pi} \quad (19)$$

With constant money supply, an equilibrium where consumption allocation and assets holdings are unchanged over time conditional on θ and where the optimal level of deposits chosen in the current period is the same as the deposits carried from the previous period (i.e. $D_{t+1}^{nd} = D_t^{nd} = D^{nd}$) will have constant prices.

When the sunspot hits, the solution to the households' problem is such that:

$$c_1^d = \frac{\bar{M}}{2p^d} \quad (20)$$

$$c_2^d = \frac{\bar{M}}{2p^d} \quad (21)$$

since the cash in advance constraint binds on both cash and credit goods and the resource constraint: $f(\frac{D^{nd}}{p^{nd}}, L) + \frac{\bar{M}}{p^d} = y + f(\frac{D^{nd}}{p^{nd}}, L)$, implies:

$$p^d = \frac{\bar{M}}{y} \quad (22)$$

Before the sunspot hits from the Euler equation we have:

$$f_1(i, L)\beta^2\left[(1 - \pi)\frac{2}{\bar{M}} + \frac{\pi}{[\bar{M} - D]}\right] = \frac{1}{[\bar{M} - D]} \quad (23)$$

that can be rearranged as

$$\begin{aligned} f_1(i, L)\beta^2(2 - \pi) - f_1(i, L)\beta^2(1 - \pi)\frac{2D}{M} &= 1 \\ \beta^2(2 - \pi)\alpha\left(\frac{D}{p}\right)^{\alpha-1}L^{1-\alpha} - 2\beta^2(1 - \pi)\phi\alpha L^{1-\alpha}\left(\frac{D}{p}\right)^{\alpha-1} &= 1 \end{aligned} \quad (24)$$

with

$$D = \phi\bar{M} \quad (25)$$

Also from the cash in advance constraint we have:

$$c_1 = \frac{\bar{M}}{p}(1 - \phi) \quad (26)$$

and from the first order conditions, using also (21), we can solve for c_2 :

$$c_2 = \frac{\bar{M}}{p} \frac{1}{2\beta(1 - \pi) + \frac{\beta\pi}{1-\phi}} \quad (27)$$

Then from the resource constraint we have:

$$c_1 + c_2 + \frac{D}{p} = y + \frac{RD}{p} \quad (28)$$

where using banks' optimality conditions $R_t = \frac{p_{t+1}}{p_t} \frac{f_1(i_t, L)}{\pi}$. Using also (26), (27) we can rewrite (28) as:

$$\begin{aligned}
\frac{\bar{M}}{p} + \frac{\bar{M}}{p} \frac{1}{2\beta(1-\pi) + \frac{\beta\pi}{1-\phi}} &= y + \frac{f_1(i, L)}{\pi} \frac{D}{p} \\
\frac{\bar{M}}{p} \left[1 + \frac{1}{2\beta(1-\pi) + \frac{\beta\pi}{1-\phi}} \right] &= y + \frac{\bar{M}}{p} \phi \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{\bar{M}\phi}{p} \right)^{\alpha-1} \\
&= y + \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{\bar{M}\phi}{p} \right)^\alpha
\end{aligned} \tag{29}$$

We can also rewrite (24) as:

$$\begin{aligned}
\beta^2(2-\pi)\alpha \left(\frac{\phi\bar{M}}{p} \right)^{\alpha-1} L^{1-\alpha} - 2\beta^2(1-\pi)\phi\alpha L^{1-\alpha} \left(\frac{\phi\bar{M}}{p} \right)^{\alpha-1} &= 1 \\
[\beta^2(2-\pi)\alpha L^{1-\alpha}\phi^{\alpha-1} - 2\beta^2(1-\pi)\alpha L^{1-\alpha}\phi^\alpha] \left(\frac{\bar{M}}{p} \right)^{\alpha-1} &= 1 \\
\left(\frac{1}{\alpha\beta^2 L^{1-\alpha}\phi^{\alpha-1} [2-\pi-2\phi(1-\pi)]} \right)^{\frac{1}{\alpha-1}} &= \frac{\bar{M}}{p}
\end{aligned} \tag{30}$$

Now we can combine (29) and (30) to obtain:

$$\begin{aligned}
&\left(\frac{1}{\alpha\beta^2 L^{1-\alpha}\phi^{\alpha-1} [2-\pi-2\phi(1-\pi)]} \right)^{\frac{1}{\alpha-1}} \left[1 + \frac{1}{2\beta(1-\pi) + \frac{\beta\pi}{1-\phi}} \right] \\
&= y + \phi^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{1}{\alpha\beta^2 L^{1-\alpha}\phi^{\alpha-1} [2-\pi-2\phi(1-\pi)]} \right)^{\frac{\alpha}{\alpha-1}} \\
&\left(\frac{1}{\alpha\beta^2 L^{1-\alpha}\phi^{\alpha-1} [2-\pi-2\phi(1-\pi)]} \right)^{\frac{1}{\alpha-1}} \left[1 + \frac{1}{2\beta(1-\pi) + \frac{\beta\pi}{1-\phi}} \right] \\
&= y + \frac{\alpha L^{1-\alpha}}{\pi} (\alpha\beta^2 L^{1-\alpha})^{\frac{\alpha}{1-\alpha}} (2-\pi-2\phi(1-\pi))^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{31}$$

Define:

$$\begin{aligned}
LHS1(\phi) &= (\alpha\beta^2 L^{1-\alpha} \phi^{\alpha-1} [2 - \pi - 2\phi(1 - \pi)])^{\frac{1}{1-\alpha}} \\
LHS2(\phi) &= \left[1 + \frac{1}{2\beta(1 - \pi) + \frac{\beta\pi}{1-\phi}}\right] \\
LHS(\phi) &= LHS1(\phi)LHS2(\phi) \\
RHS2(\phi) &= \frac{\alpha L^{1-\alpha}}{\pi} (\alpha\beta^2 L^{1-\alpha})^{\frac{\alpha}{1-\alpha}} (2 - \pi - 2\phi(1 - \pi))^{\frac{\alpha}{1-\alpha}} \\
RHS(\phi) &= y + RHS2(\phi)
\end{aligned}$$

Then:

$$\frac{\partial RHS2(\phi)}{\partial \phi} = \frac{\alpha L^{1-\alpha}}{\pi} (\alpha\beta^2 L^{1-\alpha})^{\frac{\alpha}{1-\alpha}} \frac{\alpha}{1-\alpha} (2 - \pi - 2\phi(1 - \pi))^{\frac{\alpha}{1-\alpha}-1} (-2(1 - \pi)) \quad (32)$$

$$\frac{\partial RHS(\phi)}{\partial \phi} = \frac{\partial RHS2(\phi)}{\partial \phi} \quad (33)$$

$$\begin{aligned}
\frac{\partial LHS1(\phi)}{\partial \phi} &= \frac{\alpha\beta^2 L^{1-\alpha}}{1-\alpha} (\phi^{\alpha-1} \alpha\beta^2 L^{1-\alpha} [2 - \pi - 2\phi(1 - \pi)])^{\frac{\alpha}{1-\alpha}} \\
&[(\alpha - 1)\phi^{\alpha-2} (2 - \pi - 2\phi(1 - \pi)) - 2\phi^{\alpha-1} (1 - \pi)] \quad (34)
\end{aligned}$$

$$\frac{\partial LHS2(\phi)}{\partial \phi} = -\frac{1}{[2\beta(1 - \pi) + \frac{\beta\pi}{1-\phi}]^2} \frac{\beta\pi}{(1 - \phi)^2} \quad (35)$$

$$\frac{\partial LHS(\phi)}{\partial \phi} = \frac{\partial LHS1(\phi)}{\partial \phi} LHS2(\phi) + LHS1(\phi) \frac{\partial LHS2(\phi)}{\partial \phi} \quad (36)$$

for values of $\phi \in [0, 1]$, which is equivalent to the constraint $D \in [0, \bar{M}]$ from the households' problem, and since $\pi > 0$ then by (32) $\frac{\partial RHS2(\phi)}{\partial \phi} < 0$. Therefore by (33) $\frac{\partial RHS(\phi)}{\partial \phi} < 0$.

Also:

- since $\alpha < 1$ and $\phi \in [0, 1]$ then by (34) $\frac{\partial LHS1(\phi)}{\partial \phi} < 0$:

- since $\pi > 0$ then $LHS1(\phi) > 0$;
- $\forall \phi \in [0, 1] \frac{\partial LHS2(\phi)}{\partial \phi} < 0$ and $LHS2(\phi) > 0$;
- therefore by (36) $\frac{\partial LHS(\phi)}{\partial \phi} < 0$.

And:

$$\begin{aligned} \lim_{\phi \rightarrow 0} LHS1(\phi) &= +\infty \\ LHS2(0) &= 1 + \frac{1}{2\beta(1-\pi) + \beta\pi} \\ &> 0 \\ \lim_{\phi \rightarrow 0} LHS(\phi) &= +\infty \\ RHS(0) &= y + \frac{\alpha L^{1-\alpha}}{\pi} (\alpha\beta^2 L^{1-\alpha})^{\frac{\alpha}{1-\alpha}} (2-\pi)^{\frac{\alpha}{1-\alpha}} \\ &> 0 \end{aligned}$$

Similarly:

$$\begin{aligned} LHS1(1) &= (\alpha\beta^2 L^{1-\alpha} \pi)^{\frac{1}{1-\alpha}} \\ &> 0 \\ \lim_{\phi \rightarrow 1} LHS2(\phi) &= 1 \\ \lim_{\phi \rightarrow 1} LHS(\phi) &= (\alpha\beta^2 L^{1-\alpha} \pi)^{\frac{1}{1-\alpha}} \\ RHS(1) &= y + \frac{\alpha L^{1-\alpha}}{\pi} (\alpha\beta^2 L^{1-\alpha})^{\frac{\alpha}{1-\alpha}} (\pi)^{\frac{\alpha}{1-\alpha}} \\ &> 0 \end{aligned}$$

Therefore since Assumption 4 implies that $\lim_{\phi \rightarrow 1} LHS(\phi) < RHS(1)$ and since $RHS(0) < \infty = \lim_{\phi \rightarrow 0} LHS(\phi)$, then by the intermediate value theorem $\exists \phi^{nd} \in (0, 1) : LHS(\phi^{nd}) = RHS(\phi^{nd})$, given that both $LHS(\phi)$ and $RHS(\phi)$ are continuous in ϕ on the interval $[\epsilon, 1]$, $\epsilon > 0$ arbitrarily small. Also, since both $LHS(\phi)$ and $RHS(\phi)$ are monotonic in $\phi \in (0, 1)$ by (36) and (33), then such $\phi^{nd} \in (0, 1)$ is unique. Given such $\phi^{nd} \in (0, 1)$ then we can pin down p^{nd} from (30), so that from the pair (ϕ^{nd}, p^{nd}) we can construct the time invariant allocation for any time $s < t$ with t being the first date at which the sunspot hits ($\theta_t = 1$), that solves the households' problem and satisfies market clearing conditions. Denote $(c_1^{nd}, c_2^{nd}, D^{nd}, M^{nd}, i^{nd})$ the allocation associated with (ϕ^{nd}, p^{nd}) .

For this allocation together with price p^{nd} to be an equilibrium we still need to check that banks' incentives to default are consistent with the constructed strategy. However in order to do that we want to characterize p^{nd} relative to p^d .

From (22) if $\frac{\bar{M}}{p^{nd}} < y$ then $p^{nd} > p^d$. From (30) this is equivalent to:

$$(\alpha\beta^2 L^{1-\alpha} \phi^{\alpha-1} [2 - \pi - 2\phi(1 - \pi)]^{\frac{1}{1-\alpha}} < y \quad (37)$$

Define:

$$\Phi(\phi) = (\alpha\beta^2 L^{1-\alpha} \phi^{\alpha-1} [2 - \pi - 2\phi(1 - \pi)]^{\frac{1}{1-\alpha}} \quad (38)$$

Then:

$$\frac{\partial \Phi}{\partial \phi} = \frac{1}{1-\alpha} (\alpha \beta^2 L^{1-\alpha} \phi^{\alpha-1} [2 - \pi - 2\phi(1-\pi)])^{\frac{\alpha}{1-\alpha}} \quad (39)$$

$$[(\alpha-1)\alpha \beta^2 L^{1-\alpha} \phi^{\alpha-2} [2 - \pi - 2\phi(1-\pi)] + \alpha \beta^2 L^{1-\alpha} \phi^{\alpha-1} (-2(1-\pi))] \quad (40)$$

$$< 0 \quad (41)$$

Also $\lim_{\phi \rightarrow 0} \Phi(\phi) = +\infty$, and $\Phi(1) = [\alpha \beta^2 L^{1-\alpha} \pi]^{\frac{1}{1-\alpha}}$. Using assumption 4, by the intermediate value theorem there exists a $\phi_y \in (0, 1)$ defined as follows:

$$\phi_y = \{\phi \in (0, 1) : (\alpha \beta^2 L^{1-\alpha} \phi^{\alpha-1} [2 - \pi - 2\phi(1-\pi)])^{\frac{1}{1-\alpha}} = y\} \quad (42)$$

Since $\forall \phi \in (0, 1)$, $\frac{\partial \Phi}{\partial \phi} < 0$ then ϕ_y is unique. Also, define:

$$\widehat{\Phi}(\phi) = (\alpha \beta^2 L^{1-\alpha} \phi^{-1} [2 - \pi - 2\phi(1-\pi)])^{\frac{1}{1-\alpha}} \quad (43)$$

where $\frac{\partial \widehat{\Phi}}{\partial \phi} < 0$ and $\lim_{\phi \rightarrow 0} \widehat{\Phi}(\phi) = +\infty$ therefore by assumption 4 there exists $\widehat{\phi}_y \in (0, 1)$ defined as follows:

$$\widehat{\phi}_y = \{\phi \in (0, 1) : (\alpha \beta^2 L^{1-\alpha} \phi^{-1} [2 - \pi - 2\phi(1-\pi)])^{\frac{1}{1-\alpha}} = y\} \quad (44)$$

Since $\forall \phi \in (0, 1)$, $\phi^{\alpha-1} < \phi^{-1}$ then $\widehat{\phi}_y > \phi_y$. Also, since $\frac{\partial \Phi}{\partial \phi} < 0$ by (41), if $\phi^{nd} > \phi_y$ then $p^{nd} > p^d$ because by construction ϕ_y is defined so that $\frac{\overline{M}}{p(\phi_y)} = \Phi(\phi_y) = y$ and

$p^d = \frac{\bar{M}}{y}$. Therefore, if

$$LHS1(\phi_y)LHS2(\phi_y) > y + RHS2(\phi_y) \quad (45)$$

then $\phi^{nd} > \phi_y$ and $p^{nd} > p^d$. Notice that:

$$\begin{aligned} RHS2(\phi) &= (LHS1(\phi))^\alpha \phi^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \\ &= \frac{\alpha L^{1-\alpha}}{\pi} [\alpha \beta^2 L^{1-\alpha} (2 - \pi - 2\phi(1 - \pi))]^{\frac{\alpha}{1-\alpha}} \\ LHS1(\phi) &= [\alpha \beta^2 L^{1-\alpha} (2 - \pi - 2\phi(1 - \pi)) \frac{1}{\phi^{1-\alpha}}]^{\frac{1}{1-\alpha}} \\ &= [\alpha \beta^2 L^{1-\alpha} (2 - \pi - 2\phi(1 - \pi))]^{\frac{1}{1-\alpha}} \frac{1}{\phi} \\ [LHS1(\phi)]^\alpha &= [\alpha \beta^2 L^{1-\alpha} (2 - \pi - 2\phi(1 - \pi))]^{\frac{\alpha}{1-\alpha}} \frac{1}{\phi^\alpha} \end{aligned} \quad (46)$$

then since $\frac{\partial LHS1(\phi)}{\partial \phi} < 0$, $\frac{\partial RHS2(\phi)}{\partial \phi} < 0$ and $\hat{\phi}_y > \phi_y$ then:

$$LHS2(\phi_y) > LHS2(\hat{\phi}_y) \quad (47)$$

Also ϕ^α is increasing in ϕ so

$$(\hat{\phi}_y)^\alpha > \phi_y^\alpha \quad (48)$$

Therefore if:

$$LHS1(\phi_y)LHS2(\hat{\phi}_y) > y + (LHS1(\phi_y))^\alpha (\hat{\phi}_y)^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \quad (49)$$

then also (45) is satisfied. In fact:

$$LHS1(\phi_y)LHS2(\phi_y) > LHS1(\phi_y)LHS2(\widehat{\phi}_y) \quad (50)$$

and

$$y + (LHS1(\phi_y))^\alpha (\widehat{\phi}_y)^\alpha \frac{\alpha L^{1-\alpha}}{\pi} > y + (LHS1(\phi_y))^\alpha (\phi_y)^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \quad (51)$$

Rearranging (49) yields:

$$\frac{y}{2\beta(1-\pi) + \beta\pi \frac{y^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{y^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} > \frac{\alpha L^{1-\alpha}}{\pi} y^\alpha \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{y^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha \quad (52)$$

Since Assumption 7 implies $\alpha < \pi$ and $\bar{r} > 1$ then assumption (6) is a sufficient condition for (49) to be satisfied²⁰ and since $\widehat{\phi}_y > \phi_y$ because $\widehat{\Phi}(\phi) > \Phi(\phi)$ then $LHS(\phi_y) > RHS(\phi_y)$. Since by (36) and (33) $\frac{\partial LHS(\phi)}{\partial \phi} < 0$ and $\frac{\partial RHS(\phi)}{\partial \phi} < 0$ and since $LHS(\phi^{nd}) = RHS(\phi^{nd})$ and $LHS(\phi_y) > RHS(\phi_y)$ then it must be that $\phi^{nd} > \phi_y$

²⁰In fact assumption (6) can be rearranged as:

$$\frac{y\alpha}{\pi\bar{r}} \left[1 + \frac{1}{2\beta(1-\pi) + \beta\pi \frac{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} \right] > y + y^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha$$

and since Assumption 7 implies $\alpha < \pi$ and $\bar{r} > 1$ then:

$$y + \frac{y}{2\beta(1-\pi) + \beta\pi \frac{y^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{y^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} > \frac{y\alpha}{\pi\bar{r}} \left[1 + \frac{1}{2\beta(1-\pi) + \beta\pi \frac{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} \right]$$

$$y + \frac{\alpha L^{1-\alpha}}{\pi} y^\alpha \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha > y + \frac{\alpha L^{1-\alpha}}{\pi} y^\alpha \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{y^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha$$

therefore (49) is satisfied.

and therefore $p^{nd} > p^d$. Then banks' default incentives are consistent with the constructed strategy if at prices p^{nd} and allocation $(c_1^{nd}, c_2^{nd}, D^{nd}, M^{nd}, i^{nd})$ banks do not default and at prices p^d and allocation (c_1^d, c_2^d) banks default.

At prices p^{nd} banks' assets net of liabilities are:

$$\begin{aligned}
f(i^{nd}, L) - \frac{R^{nd}D^{nd}}{p^{nd}} &= (i^{nd})^\alpha L^{1-\alpha} - \frac{\alpha}{\pi}(i^{nd})^\alpha L^{1-\alpha} \\
&= (i^{nd})^\alpha L^{1-\alpha} \left(1 - \frac{\alpha}{\pi}\right) \\
&\geq 0
\end{aligned} \tag{53}$$

where (53) holds because from Assumption 7 it follows that $\alpha < \pi$ and where $R^{nd} = \frac{\alpha(i^{nd})^{\alpha-1}L^{1-\alpha}}{\pi} = \frac{\alpha f(i^{nd}, L)}{\pi i^{nd}}$ from banks' profit maximization. Also bank j 's payoffs from defaulting and not defaulting at time s are:

$$\begin{aligned}
\delta_s(j) &= 0 : f(i^{nd}, L) - \frac{R^{nd}D^{nd}}{p^{nd}} + \gamma w^j \\
\delta_s(j) &= 1 : f(i^{nd}, L)
\end{aligned}$$

where

$$\begin{aligned}
\gamma w^j &= \gamma \left[\pi \left(f(i^{nd}, L) - \frac{R^{nd} D^{nd}}{p^{nd}} \right) + (1 - \pi) f(i^{nd}, L) + \gamma \pi \left(\pi \left(f(i^{nd}, L) - \frac{R^{nd} D^{nd}}{p^{nd}} \right) \right. \right. \\
&\quad \left. \left. + (1 - \pi) f(i^{nd}, L) \right) + \gamma^2 \pi^2 \left(\pi \left(f(i^{nd}, L) - \frac{R^{nd} D^{nd}}{p^{nd}} \right) + (1 - \pi) f(i^{nd}, L) \right) + \dots \right] \\
&= \gamma \left(f(i^{nd}, L) - \pi \frac{R^{nd} D^{nd}}{p^{nd}} \right) + \gamma^2 \pi \left(f(i^{nd}, L) - \pi \frac{R^{nd} D^{nd}}{p^{nd}} \right) + \dots \\
&= \gamma \left(f(i^{nd}, L) - \pi \frac{R^{nd} D^{nd}}{p^{nd}} \right) \frac{1}{1 - \gamma \pi} \\
&= \gamma f_2(i^{nd}, L) L \frac{1}{1 - \gamma \pi} \\
&= \gamma (1 - \alpha) f(i^{nd}, L) \frac{1}{1 - \gamma \pi} \tag{54}
\end{aligned}$$

and where if $\delta_s(j) = 1$ then bank j loses its endowment L forever after, so the continuation value is zero. Bank j chooses not to default ($\delta_s(j) = 0$) if $\frac{R^{nd} D^{nd}}{p^{nd}} < \gamma w^j$ that is to say:

$$\begin{aligned}
\frac{\alpha f(i^{nd}, L) D^{nd}}{\pi i^{nd} p^{nd}} &< \frac{\gamma (1 - \alpha) f(i^{nd}, L)}{1 - \gamma \pi} \\
\frac{\alpha f(i^{nd}, L)}{\pi i^{nd}} i^{nd} &< \frac{\gamma (1 - \alpha) f(i^{nd}, L)}{1 - \gamma \pi} \\
\frac{\alpha}{\pi} &< \frac{\gamma (1 - \alpha)}{1 - \gamma \pi} \\
\frac{\alpha (1 - \gamma \pi)}{\pi \gamma (1 - \alpha)} &< 1 \tag{55}
\end{aligned}$$

where (55) holds by assumption 5. At prices p^d instead:

$$\begin{aligned}
f(i^{nd}, L) - \frac{R^{nd}D^{nd}}{p^d} &= (i^{nd})^\alpha L^{1-\alpha} - \frac{\alpha}{\pi} (i^{nd})^\alpha L^{1-\alpha} \frac{p^{nd}}{p^d} \\
&= (i^{nd})^\alpha L^{1-\alpha} \left(1 - \frac{\alpha p^{nd}}{\pi p^d}\right) \\
&< 0
\end{aligned} \tag{56}$$

where (56) holds because $\frac{\alpha}{\pi} > \frac{p^d}{p^{nd}}$. In fact:

$$\begin{aligned}
\frac{p^d}{p^{nd}} &= \frac{\bar{M}}{y} \frac{1}{p^{nd}} \\
&= \frac{\Phi(\phi^{nd})}{y} \\
&= \frac{1}{y} (\alpha\beta^2 L^{1-\alpha} (\phi^{nd})^{\alpha-1} [2 - \pi - 2\phi^{nd}(1 - \pi)])^{\frac{1}{1-\alpha}} \\
&< \frac{\alpha}{\pi}
\end{aligned} \tag{57}$$

$$\tag{58}$$

where, in order to obtain (58) we can define:

$$\underline{\phi} = \left\{ \phi \in (0, 1) : (\alpha\beta^2 L^{1-\alpha} \phi^{\alpha-1} [2 - \pi - 2\phi(1 - \pi)])^{\frac{1}{1-\alpha}} = \frac{y\alpha}{\pi} \right\} \tag{59}$$

$$\widehat{\phi} = \left\{ \phi \in (0, 1) : (\alpha\beta^2 L^{1-\alpha} \phi^{-1} [2 - \pi - 2\phi(1 - \pi)])^{\frac{1}{1-\alpha}} = \frac{y\alpha}{\pi} \right\} \tag{60}$$

so that $\widehat{\phi} = \frac{(2-\pi)\alpha\beta^2 L^{1-\alpha}}{(\frac{y\alpha}{\pi})^{1-\alpha} + 2\alpha\beta^2 L^{1-\alpha}(1-\pi)}$ and by construction $\widehat{\phi} > \underline{\phi}$. If $\phi^{nd} > \underline{\phi}$ then since $\frac{\partial \Phi}{\partial \phi} < 0$ it must be that $\Phi(\phi^{nd}) < \Phi(\underline{\phi}) = \frac{y\alpha}{\pi}$ by definition of $\underline{\phi}$ in (60), and therefore inequality (58) holds. In order to argue that $\phi^{nd} > \underline{\phi}$ similarly to our previous

argument with ϕ_y , if:

$$LHS(\underline{\phi}) > RHS(\underline{\phi}) \quad (61)$$

then $\phi^{nd} > \underline{\phi}$. Inequality (61) can be rewritten as:

$$LHS1(\underline{\phi})LHS2(\underline{\phi}) > y + RHS2(\underline{\phi}) \quad (62)$$

$$> y + [LHS(\underline{\phi})]^\alpha \underline{\phi}^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \quad (63)$$

where (63) follows from (46). Inequality (63) can be rewritten as:

$$\frac{y\alpha}{\pi} LHS2(\underline{\phi}) > y + \left[\frac{y\alpha}{\pi}\right]^\alpha \underline{\phi}^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \quad (64)$$

Since $\frac{\partial LHS2}{\partial \phi} < 0$ and ϕ^α is a strictly increasing function of ϕ and $\widehat{\phi} > \underline{\phi}$ then:

$$\frac{y\alpha}{\pi} LHS2(\underline{\phi}) > \frac{y\alpha}{\pi} LHS2(\widehat{\phi}) \quad (65)$$

$$y + \left[\frac{y\alpha}{\pi}\right]^\alpha \widehat{\phi}^\alpha \frac{\alpha L^{1-\alpha}}{\pi} > y + \left[\frac{y\alpha}{\pi}\right]^\alpha \underline{\phi}^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \quad (66)$$

Therefore if:

$$\frac{y\alpha}{\pi} LHS2(\widehat{\phi}) > y + \left[\frac{y\alpha}{\pi}\right]^\alpha \widehat{\phi}^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \quad (67)$$

then (61) holds. Since from (60)

$$\widehat{\phi} = \frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{\left[\left(\frac{y\alpha}{\pi}\right)^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}\right]} \quad (68)$$

then (67) becomes:

$$\frac{y\alpha}{\pi} \left[1 + \frac{1}{2\beta(1-\pi) + \beta\pi \frac{\left[\frac{y\alpha}{\pi}\right]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{\left[\frac{y\alpha}{\pi}\right]^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} \right] > y + \left(\frac{y\alpha}{\pi}\right)^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{\left[\frac{y\alpha}{\pi}\right]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha \quad (69)$$

which by assumption (6) is satisfied because $\bar{r} > 1$ and because Assumption 7 implies $\alpha > \pi$. Therefore:

$$\begin{aligned} \frac{y\alpha}{\pi} \left[1 + \frac{1}{2\beta(1-\pi) + \beta\pi \frac{\left[\frac{y\alpha}{\pi}\right]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{\left[\frac{y\alpha}{\pi}\right]^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} \right] &> \frac{y\alpha}{\pi\bar{r}} \left[1 + \frac{1}{2\beta(1-\pi) + \beta\pi \frac{\left[\frac{y\alpha}{\pi\bar{r}}\right]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{\left[\frac{y\alpha}{\pi\bar{r}}\right]^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} \right] \\ y + y^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{\left[\frac{y\alpha}{\pi}\right]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha &> y + \left(\frac{y\alpha}{\pi}\right)^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{\left[\frac{y\alpha}{\pi}\right]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha \end{aligned}$$

■

A.2 Proof of Proposition 2

With an active monetary policy, when the sunspot hits, prices will never fall, regardless of the measure of defaulting banks because the Monetary Authority will inject cash on the securities' market if a positive measure of banks default, exactly in the amount necessary to maintain prices at no default level. Therefore since prices will

be kept at no default level (p^{nd}),

- with $\frac{D^{nd}}{p^{nd}} > 0$, banks are able to meet their obligations by (53) and have no incentive to default by (55).
- with $\frac{D^{nd}}{p^{nd}} = 0$, then $f(0, L) = 0$, since no investment into the productive technology could be made in the previous period without banks having access to deposits as a source of funds, and $\frac{RD^{nd}}{p^{nd}} = 0$ since banks have no liabilities²¹.

Therefore with an active monetary policy it is a strictly dominant strategy for each bank j not to default. Households keep depositing a strictly positive fraction of their assets and no banking panics occur.

A.3 Proof of Proposition 3

Proposition 3. 1 *Without deposit insurance banks invest in f*

Proof. The proof is in two steps. First we argue that the best response to banks $j' \in ([0, 1] \setminus \{j\})$ investing in the safe technology is to invest in the safe technology too, so choosing the safe technology is an equilibrium. Then we argue that choosing the risky technology is not an equilibrium: when banks $j' \in ([0, 1] \setminus \{j\})$ invest in the risky technology it is a profitable deviation for bank j to invest in the safe technology.

Claim 1 *Investing in the safe technology is an equilibrium.*

²¹Banks' payoffs from not defaulting and defaulting are the same and both equal to 0 in this case so banks are indifferent between defaulting or not and they do not default by (6). This is equivalent to assuming that by defaulting banks incur a cost $\kappa > 0$ arbitrarily small, so that when deposits are zero banks are strictly better off by not defaulting and since κ is arbitrarily small then it does not alter banks incentives to default when deposits are strictly positive.

Proof. By contradiction.

Suppose banks $j' \in ([0, 1] \setminus \{j\})$ choose to invest in the safe technology f and that investing in the risky technology \hat{f} is a profitable deviation for bank j . Bank j can either offer households the same contract as banks j' or renegotiate the contract²².

If bank j offered households the same contract as banks j' then we can argue that households would not deposit because banks would default in more states of the world. The following table summarizes bank j assets net of liabilities and its payoffs in every state of the world:

Table 1

\hat{r}	Prices	Net assets	No default	Default
$\underline{r} = 0$	p^{nd}	$-\frac{R^{nd}D^{nd}}{p^{nd}}$	$-\frac{R^{nd}D^{nd}}{p^{nd}} + \gamma w^j$	0
$\underline{r} = 0$	p^d	$-\frac{R^{nd}D^{nd}}{p^d}$	$-\frac{R^{nd}D^{nd}}{p^d} + \gamma w^j$	0
\bar{r}	p^{nd}	$\bar{r}\left(\frac{D^{nd}}{p^{nd}}\right)^\alpha L^{1-\alpha} - \frac{R^{nd}D^{nd}}{p^{nd}}$	$\bar{r}\left(\frac{D^{nd}}{p^{nd}}\right)^\alpha L^{1-\alpha} - \frac{R^{nd}D^{nd}}{p^{nd}} + \gamma w^j$	$\bar{r}\left(\frac{D^{nd}}{p^{nd}}\right)^\alpha L^{1-\alpha}$
\bar{r}	p^d	$\bar{r}\left(\frac{D^{nd}}{p^d}\right)^\alpha L^{1-\alpha} - \frac{R^{nd}D^{nd}}{p^d}$	$\bar{r}\left(\frac{D^{nd}}{p^d}\right)^\alpha L^{1-\alpha} - \frac{R^{nd}D^{nd}}{p^d} + \gamma w^j$	$\bar{r}\left(\frac{D^{nd}}{p^d}\right)^\alpha L^{1-\alpha}$

so that if $\hat{r} = 0$ bank j must default since it doesn't have enough assets to pay its liabilities. If $\hat{r} = \bar{r}$ and prices are p^{nd} then bank j does not default since its assets exceed its liabilities by (53) and the continuation value of being in business is larger than the real cost of paying depositors back by (55). If $\hat{r} = \bar{r}$ and prices are p^d then

²²By contract we mean a state contingent payment schedule for a given amount of deposits that households made. So if bank j renegotiates the contract it will offer a state contingent interest rate on the deposits previously made by households, so it will promise to pay households different interest rates in different states of the world. Recall that households however cannot change the amount of assets that they want to deposit, because once their portfolio decision is made, all they can do is moving them to a different bank after observing its choice of technology.

bank j assets net of liabilities are:

$$\begin{aligned}
\bar{r}f(i^{nd}, L) - \frac{R^{nd}D^{nd}}{p^d} &= \bar{r}(i^{nd})^\alpha L^{1-\alpha} - \frac{\alpha}{\pi}(i^{nd})^\alpha L^{1-\alpha} \frac{p^{nd}}{p^d} \\
&= (i^{nd})^\alpha L^{1-\alpha} \left(\bar{r} - \frac{\alpha p^{nd}}{\pi p^d} \right) \\
&< 0
\end{aligned} \tag{70}$$

where (70) holds because $\frac{p^d}{p^{nd}} < \frac{\alpha}{\pi} \frac{1}{\bar{r}}$. In fact by (57) and similarly to our previous argument with $\underline{\phi}$, we can define:

$$\underline{\phi}_r = \left\{ \phi \in (0, 1) : (\alpha\beta^2 L^{1-\alpha} \phi^{\alpha-1} [2 - \pi - 2\phi(1 - \pi)])^{\frac{1}{1-\alpha}} = \frac{\alpha y}{\pi \bar{r}} \right\} \tag{71}$$

$$\widehat{\phi}_r = \left\{ \phi \in (0, 1) : (\alpha\beta^2 L^{1-\alpha} \phi^{-1} [2 - \pi - 2\phi(1 - \pi)])^{\frac{1}{1-\alpha}} = \frac{\alpha y}{\pi \bar{r}} \right\} \tag{72}$$

and if $\phi^{nd} > \underline{\phi}_r$ then (70) is satisfied. We can argue that $\phi^{nd} > \underline{\phi}_r$ by showing that $LHS(\underline{\phi}_r) > RHS(\underline{\phi}_r)$. Using (46) a sufficient condition for that to be true is:

$$LHS1(\underline{\phi}_r)LHS2(\widehat{\phi}_r) > y + \widehat{\phi}_r^\alpha [LHS1(\underline{\phi}_r)]^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \tag{73}$$

$$\frac{\alpha y}{\pi \bar{r}} LHS2(\widehat{\phi}_r) > y + \widehat{\phi}_r^\alpha \left[\frac{\alpha y}{\pi \bar{r}} \right]^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \tag{74}$$

From (72):

$$\widehat{\phi}_r = \frac{\alpha\beta^2 L^{1-\alpha} (2 - \pi)}{\left[\left(\frac{\alpha y}{\pi \bar{r}} \right)^{1-\alpha} + 2(1 - \pi)\alpha\beta^2 L^{1-\alpha} \right]} \tag{75}$$

then (74) becomes:

$$\frac{y\alpha}{\pi\bar{r}} \left[1 + \frac{1}{2\beta(1-\pi) + \beta\pi \frac{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}}{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} - \pi\alpha\beta^2 L^{1-\alpha}}} \right] > y + \left(\frac{y\alpha}{\pi\bar{r}} \right)^\alpha \frac{\alpha L^{1-\alpha}}{\pi} \left(\frac{\alpha\beta^2 L^{1-\alpha}(2-\pi)}{[\frac{y\alpha}{\pi\bar{r}}]^{1-\alpha} + 2(1-\pi)\alpha\beta^2 L^{1-\alpha}} \right)^\alpha \quad (76)$$

which by assumption (6) is satisfied since $\alpha < \pi$ and $\bar{r} > 1$. Therefore $\phi^{nd} > \underline{\phi}_r$ and (70) holds, so that bank j defaults when $\hat{r} = \bar{r}$ and prices are p^d . Since bank j defaults more often than banks j' then households will not deposit in bank j . Therefore investing in the risky technology \hat{f} is not a profitable deviation for bank j and investing in the safe technology f is an equilibrium. ■

Claim 2 *There is no equilibrium where banks invest in the risky technology.*

Proof. By contradiction.

Suppose a measure one of banks chooses to invest in the risky technology \hat{f} and that there exists a sequence of allocations that solves the households' problem and clears the markets²³ for a given interest rate on deposits²⁴ $R_s = \frac{p_{s+1}}{p_s} \bar{r} \alpha \left(\frac{D_s}{p_s} \right)^{\alpha-1} L^{1-\alpha}$.

²³When the economy starts from initial level of deposits D^{nd} but banks invest in a different technology, unless they offer the same deposit contract as if they invested in the safe technology, the interest rate associated with a competitive deposit contract is $R_s \neq R^{nd}$ paid in different states of the world. Therefore unless we show that there exist a sequence of non time invariant allocations and prices that solve the households' problem, banks' problem and clear the markets, an equilibrium may not exist. However here we want to claim that an equilibrium with banks investing in \hat{f} does not exist, so we only need to rule out that investing in \hat{f} is a best response even assuming that a solution to the households' problem and market clearing conditions exists.

²⁴If a measure one of banks invests in the risky technology \hat{f} then the equilibrium interest rate on the deposit contract they offer is derived from the optimality conditions to the following problem:

$$\begin{aligned} \max_{i_s, D_s^b} & \quad [q\pi(\bar{r}i_s^\alpha L^{1-\alpha} - \frac{R_s D_s^b}{p_{s+1}}) + q(1-\pi)\bar{r}i_s^\alpha L^{1-\alpha}] \\ \text{s.t.} & \quad p_s i_s \leq D_s^b \end{aligned}$$

Suppose that investing in \hat{f} is a best response for bank j . Then it is possible to construct a profitable deviation for bank j as follows:

- invest in the safe technology f today and then follow the same strategy played by all other banks from tomorrow onwards,
- offer households the same contract as other banks offer.

As long as Assumption 7 holds then it is always feasible for bank j to pay the same interest rate as in the contract offered by banks $j' \in ([0, 1] \setminus \{j\})$. In fact bank j assets net of liabilities would be:

$$\begin{aligned} i_s^\alpha L^{1-\alpha} - \frac{p_{s+1}}{p_s} \frac{\bar{r}}{\pi} \alpha \left(\frac{D_s}{p_s}\right)^{\alpha-1} L^{1-\alpha} \frac{D_s}{p_s} &= \left(\frac{D_s}{p_s}\right)^\alpha L^{1-\alpha} - \frac{p_{s+1}}{p_s} \frac{\bar{r}}{\pi} \alpha \left(\frac{D_s}{p_s}\right)^{\alpha-1} L^{1-\alpha} \frac{D_s}{p_s} \\ &= \left(1 - \frac{\bar{r}\alpha}{\pi}\right) \left(\frac{D_s}{p_s}\right)^\alpha L^{1-\alpha} \end{aligned}$$

Then the expected continuation payoff of being in business for bank j is the same as for every bank $j' \in ([0, 1] \setminus \{j\})$, whatever choice of technology they all will follow from tomorrow onwards, because the contract they all offer is the same. Letting $w_{s+1}^j = w_{s+1}^{j'}$ denote the continuation payoff of still being in business at the end of date s , then the total expected payoff for bank j from:

since banks behave competitively and take prices as given. Recall that banks default in all states of the world but when $\hat{r} = \bar{r}$ and $p_{s+1} = p^{nd}$ by (70) and Table 1 and then since $\underline{r} = 0$ then banks have zero assets when $\hat{r} = \underline{r}$.

- investing in \widehat{f} is:

$$\begin{aligned}
& \pi\left[\alpha\left(\bar{r}f(i_s, L) - \frac{R_s D_s}{p_{s+1}}\right) + (1 - \alpha)\left(\bar{r}f(i_s, L) - \frac{R_s D_s}{p_{s+1}}\right)\right] \\
& + (1 - \pi)\left[\alpha\bar{r}f(i_s, L) + (1 - \alpha)r\underline{f}(i_s, L)\right] + \gamma w_{s+1}^j \\
& = \pi[\alpha\bar{r} + (1 - \alpha)r\underline{f}]f(i_s, L) - \pi\frac{R_s D_s}{p_{s+1}} + \gamma w_{s+1}^j \\
& = [\alpha\bar{r} + (1 - \alpha)r\underline{f}]f(i_s, L) - \pi\frac{R_s D_s}{p_{s+1}} + \gamma w_{s+1}^j
\end{aligned}$$

- investing in f is:

$$\pi\left[f(i_s, L) - \frac{R_s D_s}{p_{s+1}}\right] + (1 - \pi)f(i_s, L) + \gamma w_{s+1}^j = f(i_s, L) - \pi\frac{R_s D_s}{p_{s+1}} + \gamma w_{s+1}^j$$

where $\alpha\bar{r} + (1 - \alpha)r\underline{f} < 1$ since \widehat{f} is a mean reducing spread of f . Therefore the expected return from \widehat{f} is strictly smaller than the expected return from f and it is a profitable deviation for bank j to invest in the safe technology f , so that investing in the risky technology is not an equilibrium. ■

Therefore without deposit insurance it is a strictly dominant strategy for banks to invest in the safe technology. In environments without deposit insurance the results of Proposition 1 and 2 go through even banks were allowed to choose to invest between the safe and the risky technology. ■

Proposition 3.2 *With strict regulations deposit insurance can stop a panic-default*

Proof. In an environment with strict regulation the deposit insurer can force banks to invest in the safe technology, and it takes on banks liabilities when banks default besides seizing their assets. Therefore banks invest in the safe technology f and if at time t the sunspot hits and a measure one of banks defaults ($\lambda_t = \int_0^1 \delta_t(j) dj = 1$), the real value of assets seized by the deposit insurer is $f(i^{nd}, L) = r(i^{nd})^\alpha L^{1-\alpha}$. The real value of the liabilities the insurer takes on is $\frac{R^{nd}D^{nd}}{p_t}$. Since liabilities are backed up by the deposit insurer then they are still a viable means of payment for households, who can still use them to purchase credit goods exactly as if banks did not default. The deposit insurer in fact guarantees that households will get paid in the amount they were promised when they signed the deposit contract so, taking prices as given, the solution to the households' problem is the as in a no default state.

When banks default the deposit insurer seizes banks' assets, pays banks' liabilities and rebates back to banks any left over assets. Then banks' consumption at time t as a function of the price level p_t , is $c_t^b(p_t) = f(i^{nd}, L) - \frac{R^{nd}D^{nd}}{p_t}$ and the resource constraint is:

$$\begin{aligned} c_t^b(p_t) + c_{1t}(p_t) + c_{2t}(p_t) + i_t(p_t) &= y + f(i^{nd}, L) \\ f(i^{nd}, L) - \frac{R^{nd}D^{nd}}{p_t} + c_{1t}(p_t) + c_{2t}(p_t) + i_t(p_t) &= y + f(i^{nd}, L) \end{aligned}$$

Since taking prices as given the solution to the households' problem is the same as in no default then:

$$c_{1t}(p_t) = c_1^{nd}$$

$$c_{2t}(p_t) = c_2^{nd}$$

$$i_t(p_t) = i^{nd}$$

and therefore in equilibrium $p_t = p^{nd}$. This implies that even if a measure one of banks defaults, since equilibrium prices are p^{nd} then bank j would not default because its assets exceed its liabilities by (53) and its continuation value of being in business is larger than the real cost of paying depositors back by (55). Therefore with deposit insurance and strict regulations it is a strictly dominant strategy for banks not to default and banking panics do not occur. ■

Proposition 3.3 *Without strict regulations, deposit insurance can stop a panic-default but aggregate output fluctuates*

Proof. Deposit insurance by construction is designed so that the insurer seizes banks' assets if they fail, and levies lump sum taxes on households' endowment if the value of banks' assets is not sufficient to cover payments to households who hold claims at the bank. Therefore households are always paid the return on their deposits, so that banking panics do not occur.

When deposit insurance is not coupled with strict regulatory arrangements however, banks may choose to invest in the risky technology. Banks choose which tech-

nology to invest in by maximizing their expected payoff. Bank j expected payoff when deposit insurance is established at time t is:

- if investing in \widehat{f} :

$$\begin{aligned} w_t^j(\widehat{f}) &= q[\max(\bar{r}i_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t}, 0) + \gamma w_{t+1}^j] + (1-q)[\max(\underline{r} - \frac{R_{t-1}D_{t-1}}{p_t}, 0)] \\ &= q[\bar{r}i_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t} + \gamma w_{t+1}^j] \end{aligned}$$

- if investing in f :

$$\begin{aligned} w_t^j(f) &= [\max(ri_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t}, 0) + \gamma w_{t+1}^j] \\ &= [ri_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t} + \gamma w_{t+1}^j] \end{aligned}$$

where:

- the interest rate R_{t-1} on existing deposits D_{t-1} is the same regardless of whether banks choose \widehat{f} or f because by construction deposit insurance is established after households' deposit decision and before banks' investment decision.
- the continuation value w_t^j associated with investing in \widehat{f} is the same as the one associated with f because if deposit insurance is established at time t it will not be available at time $t+1$ since we are carrying out a one time experiment. Therefore from time $t+1$ onwards banks will invest in the safe technology by

Proposition refprop3 part 1, regardless of the technology they choose at time t .

Also, the continuation value w_t^j is the same as the one defined in (54) because if a default does not occur households optimal level of deposits is D^{nd} given that the payments that households receive for previously made deposits is $R^{nd}D^{nd}$.

- $\max(\bar{r}i_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t}, 0) = \bar{r}i_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t}$ because when banks' liabilities are insured, they are still a viable means of payment even if banks defaulted. Therefore prices never fall and banks do not default, since $\bar{r} > 1$ and by (53) banks' assets exceed their liabilities and by (55) the continuation value of being in business is larger than the real cost of paying depositors back. A similar argument applies for $\max(ri_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t}, 0) = ri_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t}$.
- $\max(\underline{r} - \frac{R_{t-1}D_{t-1}}{p_t}, 0) = 0$ since by normalization $\underline{r} = 0$ so that banks' assets are not sufficient to cover banks' liabilities.

Banks choose the technology they want to invest in by comparing the expected payoff from investing in the risky technology \hat{f} with the expected payoff from investing in the safe technology f . Then banks choose the risky technology \hat{f} if:

$$\begin{aligned}
q[\bar{r}i_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t} + \gamma w_{t+1}^j] &> [i_{t-1}^\alpha L^{1-\alpha} - \frac{R_{t-1}D_{t-1}}{p_t} + \gamma w_{t+1}^j] \\
q[\bar{r}(i^{nd})^\alpha L^{1-\alpha} - \frac{\alpha}{\pi}(i^{nd})^\alpha L^{1-\alpha} + \frac{\gamma(1-\alpha)}{(1-\gamma\pi)}(i^{nd})^\alpha L^{1-\alpha}] &> (i^{nd})^\alpha L^{1-\alpha} - \frac{\alpha}{\pi}(i^{nd})^\alpha L^{1-\alpha} + \\
&\quad \frac{q\gamma(1-\alpha)}{(1-\gamma\pi)}(i^{nd})^\alpha L^{1-\alpha} \\
q[\bar{r} - \frac{\alpha}{\pi} + \frac{\gamma(1-\alpha)}{(1-\gamma\pi)}] &> [1 - \frac{\alpha}{\pi} + \frac{\gamma(1-\alpha)}{(1-\gamma\pi)}] \quad (77)
\end{aligned}$$

where (77) is satisfied $\forall q > \underline{q}$ by Assumption 8.

■

A.4 Banks impatient enough

Banks expected profits maximization (8) can be rewritten as follows:

$$w_{t+1}^j(h_1^t) = \max_{\{s_t^{b,j}, D_t^{b,j}\}} x_t^{b,j}(h_1^t) + \gamma[f(i_t^j, L) - \pi \frac{R_t D_t^{b,j}}{p_{t+1}}] \quad (78)$$

$$s.t. \quad x_t^{b,j} + s_t^{b,j} = f(i_{t-1}^{b,j}, L) - \frac{R_t D_t}{p_t}$$

$$p_t i_t^j = D_t^b + p_t s_t^{b,j}$$

$$s_t^{b,j} \geq 0 \quad (79)$$

where $x_t^{b,j}(h_1^t)$ denotes time t consumption of bank j if observed history is h_1^t ; $s_t^{b,j}$ denotes the amount of time t profits that bank j saves and invest into the productive technology; i_t^j denotes actual investment by bank j into the productive technology.

Then:

Claim 3 *If Assumption 3 is satisfied then a solution to problem (78) is such that $s_t^{b,j} = 0$.*

Proof. First order conditions to problem (78) are:

$$-1 + \gamma \alpha (i_t^j)^{\alpha-1} L^{1-\alpha} \leq 0 \quad (80)$$

$$s_t^{b,j} [-1 + \gamma \alpha (i_t^j)^{\alpha-1} L^{1-\alpha}] = 0 \quad (81)$$

$$\frac{p_{t+1}}{p_t} \frac{\gamma}{\pi} f_1(i_t^j, L) = R_t \quad (82)$$

If

$$\gamma\alpha(i_t^j)^{\alpha-1}L^{1-\alpha} < 1 \quad (83)$$

then $s_t^{b,j} = 0$. Let i_t denote aggregate investment by every bank $j \in [0, 1]$, then we can rewrite the Euler equation (24) in the households problem when $i_t > \frac{D_t}{p_t}$ as:

$$i = [\alpha\beta^2L^{1-\alpha}(2 - \pi - 2\phi(1 - \pi))]^{\frac{1}{1-\alpha}} \quad (84)$$

For any interior choice of deposits by households ($\phi \in (0, 1)$) (83) becomes:

$$\frac{\gamma\alpha L^{1-\alpha}}{\alpha\beta^2L^{1-\alpha}(2 - \pi - 2\phi(1 - \pi))} < 1 \quad (85)$$

then $s_t^{b,j} = 0$. Notice that for any interior choice of deposits by households if

$$\gamma < \beta^2 \lim_{\phi \rightarrow 1} (2 - \pi - 2\phi(1 - \pi)) \quad (86)$$

$$= \beta^2\pi \quad (87)$$

$$< \beta^2(2 - \pi - 2\phi'(1 - \pi)) \quad \forall \phi' \in (0, 1) \quad (88)$$

then under Assumption 3 (83) is satisfied and $s_t^{b,j} = 0$. Therefore under Assumption 3 banks expected profits maximization problem can be written as (8) where the constraint set is simply (9). ■

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