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Chapter 1

Portfolio Theory

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1. Introduction

Consider a consumer with a given amount of income. Such a consumer typically faces two important economic decisions. First, how to allocate his or her current consumption among goods and services. Second, how to invest among various assets. These two interrelated consumer or household problems are known as the consumption-saving decision and the portfolio selection decision.

Beginning with Adam Smith, economists have systematically studied the first decision. Arguing that a consumer will choose commodities and services that offer the greatest marginal utility relative to price, a theory of value was developed that combines subjective notions from consumer utility with objective notions from the production theory of the firm. By the beginning of the twentieth century, neoclassical economists had developed a static theory of consumer behavior as part of an analysis of market pricing under conditions of perfect competition and certainty.

The asset allocation decision was not adequately addressed by neoclassical economists, probably because they treated savings as the supply of loanable funds in developing a theory of interest rate determination instead of portfolio selection. More importantly, however, these two decisions, although closely interrelated, require substantially different methodologies. The methodology of deterministic calculus is adequate for the decision of maximizing a consumer's utility subject to a budget constraint. Portfolio selection involves making a decision under uncertainty. The probabilistic notions of expected return and risk become very important. Neoclassical economists did not have such a methodology available to them and despite some very early attempts by probabilists, like Bernoulli [1738] to define and measure risk, or Irving Fisher [1906] to describe asset returns in terms of a probability distribution, the twin concepts of expected return and risk had not yet been fully integrated. An early and important attempt to do that was made by

Marschak [1938] who expressed preferences for investment by indifference curves in the mean–variance space.[1]

The methodological breakthrough of treating axiomatically the theory of choice under uncertainty was offered by von Neumann & Morgenstern [1947] and it was only a few years later that Markowitz [1952, 1959] and Tobin [1958], used this theory to formulate and solve the portfolio selection problem.

In this essay we plan to exposit portfolio theory with a special emphasis on its historical evolution and methodological foundations. In Section 2, we describe the early work of Markowitz [1952, 1959] and Tobin [1958] to illustrate the individual contributions of these authors. Following these general remarks about the early beginning of portfolio theory, we define and solve the mean–variance portfolio problem in Section 3 and relate it to its most famous intellectual first fruits, namely the two-fund separation and the capital asset pricing theory of Sharpe [1964] and Lintner [1965] in Sections 4, 5 and 6. In particular, a portion of Section 6 is devoted to the presentation of Roll's [1977] critique of the asset pricing theory's tests and the interplay of analysis and empirical testing. This leads to an analysis of the foundational assumptions of portfolio theory with respect to investor preferences and asset return distributions, both reviewed in Section 7. The contrast of methodologies is illustrated in Sections 8 and 9 where stochastic calculus and stochastic control techniques are used to generalize the consumption-investment problem to an arbitrary number of periods. Market imperfections are addressed in Section 10. The last section identifies several extensions and refers the reader to several articles, some included in this volume. It also contains our summary and conclusions.

2. The early contributions

Markowitz [1952] marks the beginning of modern portfolio theory, where for the first time, the problem of portfolio selection is clearly formulated and solved. Earlier contributions of Keynes [1936], Marschak [1938] and others only tangentially analyze investment decisions. Markowitz's focus is the explanation of the phenomenon of portfolio diversification.

Before Markowitz could propose the "expected returns–variance of returns" rule, he first had to discredit the then widely accepted principle that an investor chooses a portfolio by selecting securities that maximize discounted expected returns.[2] Markowitz points out that if an investor follows this rule, his or her

[1] Marschak [1938, p. 312] recognizes that "the unsatisfactory state of Monetary Theory as compared with General Economics is due to the fact that the principle of determinateness so well established by Walras and Pareto for the world of perishable consumption goods and labor services has never been applied with much consistency to durable goods and, still less, to claims (securities, loans, cash)". In our modern terminology we could replace the names Monetary Theory and General Economics with Financial Economics and Microeconomic Theory, respectively.

[2] Markowitz refers the reader to a standard investments textbook by Williams [1938] that elaborates the notion that portfolio choice is guided by the rule of maximizing the discounted