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# Chapter 7

# A Survey of Computational Geometry

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## 1. Introduction

Computational geometry takes an algorithmic approach to the study of geometrical problems. The principal motivation in this study is a quest for 'good' algorithms for solving geometrical problems. Of course, several practical and aesthetic factors determine what one means by a good algorithm, but the general trend has been to associate 'goodness' with the asymptotic efficiency of an algorithm in terms of the time and space complexity. Lately, however, the implementational ease and robustness also are becoming increasingly important considerations in the algorithm design.

Although many geometrical problems and algorithms were known before, computational geometry evolved into a cohesive discipline only in the mid to late seventies. An important event in this development was the publication of the Ph.D. thesis of M. Shamos [1978] in 1978. During its first decade, the field of computational geometry grew enormously as its fundamental structures were applied to a vast variety of problems in diverse disciplines, and many new tools and techniques were developed. In the process, new insights were gained into inter-relationships among some of these fundamental structures, which also led to a unification and consolidation of several disparate sets of ideas. In the last five or so years, the field has matured significantly, both in mathematical depth as well as in algorithmic ideas.

Computational geometry has had strong interactions with other fields, in mathematics as well as in applied computer science. A particularly fruitful interplay has taken place between computational geometry and combinatorial geometry. The latter is a branch of mathematics concerned primarily with the 'counting' of certain geometric structures. Examples include counting the maximum possible number of incidences between a set of lines and a set of points, and counting the number of lines that bisect a set of points. Both fields seem to have benefited from each other: combinatorial bounds for certain structures have been obtained by analyzing an algorithm that enumerates them and, conversely, the analysis of algorithms often depends crucially on the combinatorial bound on some geometric objects.

The field of computational geometry has also benefited from its interactions with other disciplines within computer science such as VLSI, database theory, robotics, computer vision, computer graphics, pattern recognition and learning theory. These areas offer a rich variety of problems that are inherently geometrical.

Due to its interconnections with many applications areas, the variety of problems studied in computational geometry is truly enormous. Our goal in this paper is quite modest: we survey the state-of-the-art in some selected areas of computational geometry, with a strong bias towards problems with an optimization component. In the process, we also hope to acquaint the reader with some of the fundamental techniques and structures in computational geometry.

Our paper has seven main sections. The survey proper begins in Section 3, while Section 2 introduces some foundational material. In particular, we briefly describe five key concepts and fundamental structures that permeate much of computational geometry, and therefore are somewhat essential to a proper understanding of the material in later sections. The structures covered are convex hulls, arrangements, geometric duality, Voronoi diagram, and point location data structures. The main body of our survey begins with Section 3, where we describe four popular geometric graphs: minimum and maximum spanning trees, relative neighborhood graphs, and Gabriel graphs. Section 4 is devoted to algorithms in path planning. The topic of path planning is a vast one, with problems ranging from finding shortest paths in a discrete graph to deciding the feasible motion of a complex robot in an environment full of complex obstacles. We briefly mention most of the major developments in path planning research over the last two decades, but to a large extent limit ourselves to issues related to shortest paths in a planar domain. In Section 5, we discuss the matching and the traveling salesman type problems in computational geometry. Section 6 describes results on a variety of problems related to shape analysis and pattern recognition. We close with some concluding remarks in Section 7. In each section, we also pose what in our opinion are the most important and interesting open problems on the topic. There are altogether twenty open problems in this survey.

## 2. Fundamental structures

### 2.1. Convex hulls

The *convex hull* of a finite set of points $S$ is the smallest convex set containing $S$. In two dimensions, for instance, the convex hull is the smallest convex polygon containing all the points of $S$; see Figure 1 for an example. In higher dimensions, the convex hull is a polytope. Before we discuss the algorithms for computing a convex hull, we must address the question of representing it. There are several representations of a convex hull, depending upon how many features of the corresponding polytope are described. In the simplest representation, we may only