

Chapter 19

Models of Auctions and Competitive Bidding

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1. Introduction

1.1. Why auctions?

There are many different ways a society can allocate and transfer goods and services.¹ Governments often rely upon economic competition for this purpose. As opposed to common transfer methods such as negotiations and posted prices, auctions and competitive bidding are particular *formalized* forms of economic competition. This formality is of special interest in the public sector and in other situations where there may not be sufficient trust or coincidence between the goals of the selling or purchasing agent and those of the owner or buyer. It also serves a need for fairness and for avoiding even the appearance of prejudice. Since the term 'bidding' is sometimes used when what is meant is deregulation with reliance upon market forces, it is worth emphasizing that bidding is far from the only form of competition and does not, in general, have a convincing claim to being the most efficient form.

The formal nature of auctions and bidding is also of special interest to analysts including operations researchers.² Those interested in the general topic of decision-making in the face of both competition and uncertainty have found auctions and bidding a fruitful area of study precisely because of the added structure imposed by the formality as well as because of the availability of added data (e.g. on unsuccessful bids).

1.2. Types of auctions

There are many different forms of auctions and bidding and several useful ways of classifying these variants. An obvious, but relatively unimportant, distinction

¹ See Shubik [1970] for an encompassing discussion of the many alternatives that have been used.

² Indeed, the first Ph.D. in operations research was granted to a student whose dissertation was on competitive bidding [Friedman, 1957]).

is between high-bid-wins auctions, typically involving bidding by potential buyers, and low-bid-wins auctions, typically involving bidding by potential sellers. A more significant distinction is between 'first-price' and 'second-price' auctions. Standard sealed bidding is a closed first-price procedure. The phrase 'first-price' means that the winning bidder pays or gets paid the amount of his or her bid, the best price. In sealed second-price bidding, an unusual procedure³ sometimes called a Vickrey auction after the economist who first proposed its use and analyzed it [Vickrey, 1961], the bidder offering the best price wins but pays (or is paid) the amount of the second best bid. For example, if three bidders competing to buy a single item make bids of \$75, \$90 and \$100, then the bidder who offered \$100 wins but pays only \$90. If there is to be more than one winner in such an auction, the auction is 'nondiscriminatory', and the price for each winner is set by the best losing bid. If two identical items were to be sold by this method to the bidders in the previous example, the bidder offering \$90 would also win, and each winner would pay \$75. On the other hand, standard sealed bidding is normally discriminatory when multiple identical objects are sold; in other words, each bidder pays the amount he or she bid.

For each of these sealed bid procedures, there is an oral procedure that is at least somewhat analogous with respect to bidding strategy and results. The analog for sealed bidding is the 'Dutch auction' in which a selling auctioneer (or, in the Dutch flower auctions, a mechanical device) announces successively lower prices until a bidder makes a bid. That bidder then becomes the winner at the price he or she offered. As with standard sealed bidding, bidders have incentive to behave strategically, i.e. to trade off the chance of losing the auction against extra profit if they win.

In some ways, the common progressive (or 'English') auction is analogous to the Vickrey auction if one conceives of the strategies of bidders in oral auctions as limits up to which they will bid. In the progressive oral auction, under commonly met assumptions, each bidder has a dominant strategy⁴ of bidding up to exactly the amount he or she values the object for sale.⁵ If all bidders do this, then the object will always be sold to the bidder valuing it most. However, if it is to be analogous strategically to the English auction, the Vickrey auction requires bidders to reveal their true valuations and to trust the bid taker. The analogy breaks down if bidders are resistant to revealing their valuations or if they fear cheating by the bid taker [see Rothkopf, Teisberg & Kahn, 1990; Engelbrecht-Wiggans & Kahn, 1991].

Bidding may be on total price or, especially when the quantity involved is not known with certainty at the time the winning competitor is selected, on one or more unit prices. This distinction can become important when the winning bidder can exercise some control over the quantity or when there are multiple unit prices involved with the winner being selected based upon an estimate of quantities.

³ See Rothkopf, Teisberg & Kahn [1990] for a discussion of the usage of this kind of auction and the reasons for it being unusual.

⁴ A dominant strategy is one that it pays to follow irrespective of what competitors do.

⁵ If the bidder bids more and wins or bids less and loses, he or she will regret it.