

Chapter 1

Single-Product, Single-Location Models

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1. Introduction

Inventory management is one area in which operations research has had a significant impact. At the present time, the total value of inventories in the United States is close to one trillion dollars (Economic Report of the President, February, 1989). Mathematical models form the basis for many of the inventory control systems in use today, whether they be inventories of raw materials, spare parts, cash, or finished goods.

Most mathematical inventory models are designed to address two fundamental issues: when should a replenishment order be placed, and how much should the order quantity be. The complexity of the model depends on the assumptions that one makes about the demand, cost structure, and physical characteristics of the system. In some cases, the operating characteristics of a given policy are analyzed.

The objective of virtually all inventory control models is to minimize costs. In most cases, minimizing costs will result in the same control policy as that obtained by maximizing profits. When uncertainty is present the traditional approach has been to minimize expected costs. The use of the expected value operator is justified from the law of large numbers: as the number of planning periods grows, the arithmetic average of the actual costs incurred will be close to the expected costs. Since most inventory control problems are ongoing, this approach makes sense.

Most inventory control problems in the real world involve multiple products. For example, spare parts systems for military applications may require management of hundreds of thousands of different items. However, it is often true that single product models are able to capture the essential elements of the problem, so that it is not necessary to explicitly include the interaction of different items into the formulation. Furthermore, multiple product models are

often too unwieldy to be of much use when the number of products is very large. For this reason, single product models dominate the literature and are used most frequently in practice.

This chapter is concerned with mathematical models for controlling the inventory of a single product. Other chapters in this volume will be concerned with multiple products and special topics.

1.1. History

Although operations research methodology is based on mathematical developments spanning centuries, much of the interest in the application of this methodology had its origins in the logistic support activities which took place during the Second World War. Interdisciplinary teams of scientists would brainstorm problems that were not amenable to traditional analysis. The success of these activities, along with the subsequent discovery of both the simplex method and the electronic computer led to the birth of operations research as a discipline. It is interesting to note that the first work on inventory modeling pre-dates these developments by about thirty years.

Ford Harris [1913] is generally credited with the discovery of the original EOQ (Economic Order Quantity, to be described in Section 2) model. Harris' analysis was the basis for an inventory system that he marketed. Interest in academic circles was sparked by the later studies of R.H. Wilson [1934]. Few papers were published in the next fifteen years, although some interest in mathematical inventory models resurfaced during the war. Landmark papers were published in the early fifties [Arrow, Harris & Marshak, 1951; Dvoretzky, Kiefer & Wolfowitz, 1952a,b] which laid the groundwork for later developments in the mathematical theory of inventories. The monograph by Whitin [1957] was also an important development in presenting the relationship of inventory management issues and classical economic thinking, and was one of the first treatments of the (Q, r) model under uncertainty (to be described in Section 5), which later became the cornerstone for many commercial inventory systems.

A number of important scholars turned their attention to mathematical inventory models during the 1950s. Bellman, Glicksberg & Gross [1955] showed how the methods of dynamic programming, a term apparently coined by the first author for describing a class of sequential decision problems, could be used to obtain structural results for a simple version of the periodic review stochastic demand problem. A collection of highly sophisticated mathematical models by Arrow, Karlin & Scarf [1958] provided much of the impetus for later work in this area. At about the same time, Wagner & Whitin [1958] discovered the solution to the dynamic lot sizing problem subject to time varying demand.

The number of papers published today on single product inventory problems is well into the thousands. These papers appear in journals published in the United States including *Management Science*, *Operations Research*, *AIIE Transactions*, *Naval Research Logistics Quarterly*, *Mathematics of Operations Research*, and *Decision Sciences*, to name a few, and many more international