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Chapter 13

Reliability and Maintainability*

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1. Introduction

Reliability theory deals with probabilistic and statistical problems which involve life distributions of components or systems. The mathematical models in reliability theory are directed toward the description of the functioning of the components or systems. These models can be used for solving problems in optimizing the survival or the mean life of the devices or systems. The study of maintenance policies, such as replacement and repair of the systems, is a part of reliability theory. The statistical aspect of reliability theory is directed toward the solutions of problems in predicting and estimating the life distribution or the mean life of the components or systems. Reliability theory has developed into an important topic for workers in many areas, especially in the engineering and biomedical sciences.

In this chapter we give a brief description of the various aspects of reliability theory. The reader should be aware that only a fraction of the important topics in reliability theory can be covered in such a brief overview. It is hoped, however, that the basic ideas of this theory can be grasped from the review.

In Section 2 we describe reliability systems. The interest there is how to model the state of a system as a function of the states of the components. We go from the simplest case of the states being only 'up' and 'down' to the most general case where the state space is a continuum.

In Section 3 we list various measures of importance in reliability theory. These include reliability and availability measures. The computational ideas, which are involved in deriving expressions for these measures, are given and are illustrated for some particular repairable and nonrepairable systems.

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Performability measures, which can be used for indicating how well a system works, are discussed in Section 4. Again, the computational aspects of these measures are illustrated for some systems that arise often in applications.

An important concept in reliability theory is the one of the 'hazard' (or failure) rate. It is useful for modeling purposes and also for characterizing distributions by their 'aging' properties. In Section 5 we discuss the notion of the hazard rate and, in particular, characterize distributions with increasing failure rate. Other aging notions, such as 'new better than used', are also described in Section 5. The use of these aging notions is illustrated by some probability inequalities which they imply. These aging notions are also useful for obtaining optimal maintenance policies. In Section 6 we describe some basic replacement policies and show how aging properties can influence the choice of an optimal replacement policy. Some repair policies such as minimal repair and imperfect repair are also described in Section 6.

The notion of the hazard rate can be used also for describing the probabilistic dependence of lives of different components. In Section 7 we describe how to do it. We show how the basic tool of multivariate conditional hazard rate functions can be used to model the joint distribution of dependent components. As an application we give a model of multivariate imperfect repair policy. Other replacement policies and models of random environment (which influences all the components) are also discussed in Section 7. A particular application of the ideas of Section 7 is the analysis of systems with repairable components. This analysis is described in Section 8.

The aging notions such as 'increasing failure rate' and 'new better than used' can be useful also in the case when there are more than one component and the lifetimes of the components are dependent. In Section 9 we discuss various extensions of such aging notions (which originally were designed for univariate lifetimes) to the multivariate setting.

Finally, in Section 10, we describe two areas of statistical inference in reliability theory. Some methods of estimating aging distribution functions are described. Also various hypothesis testing procedures for testing the hypothesis of 'no aging' (versus some 'aging' alternative such as 'increasing failure rate' or 'new better than used') are given in the first part of Section 10. The second part of Section 10 consists of a description of some accelerated life testing procedures.

2. Reliability systems: Monotonicity and coherency

A reliability system is composed of components that are subjected to degradation and failure. Any study of the performance of a reliability system requires that we relate the state of the system to the states of the components. Throughout this section we will focus our attention on the relationship between the system state and the components state at a fixed time epoch. We will also assume that the state of the system at any time epoch is fully characterized by