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Chapter X

Multiple Criteria Decision Making: Five Basic Concepts

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1. Introduction

Ever since Adam, human beings have been confronted with multiple criteria decision making problems (MCDM). Our food should taste good, smell good, look good and be nutritious. We want to have a good life, which may mean more wealth, more power, more respect and more time for ourselves, together with good health and a good second generation, etc. Indeed, in the records of human culture, all important political, economical and cultural events have involved multiple criteria in their evolutions.

Although analysis of multicriteria problems has been scattered in our human history, putting this analysis into a formal mathematical setting is fairly new. Although there are a number of scholars and scientists who have made contributions in this mathematical analysis of MCDM, Pareto [30] is perhaps one of the most recognized pioneers in this analysis. Interested readers are referred to [38, 39] for some historical notes.

The purpose of this article is not to report all significant events of who did what in the history, because that would almost surely be impossible. Rather, we shall sketch some important aspects of MCDM so that the reader can have a general picture of MCDM and be able to find the related literature for their further study.

We start with preferences as binary relations (Section 2) in which the reader can learn how mathematical functional relations are extended to binary relations to represent revealed preferences and learn some solution concepts of MCDM. Then we will present a class of MCDM methods which model human goal seeking behaviors (Section 3). Satisficing solutions, compromise solutions and goal programming will be sketched in this section. Numerical ordering is one of our great human inventions. There is a great interest in representing the preferences in terms of real valued functions or numerical orderings. This will be sketched in Section 4. Certainly, not all revealed preferences can be represented by numerical orderings. We use Section 5 to introduce domination structures and their related solutions to tackle the problems of representing such revealed preferences and locating good solutions. As linear functions are the best and simplest functions in mathematical analysis, linear cases always yield special results. We treat linear cases and their related topics including the MC and MC^2 simplex method in Section 6. Finally, we offer some further comments in Section 7 as a summary and conclusion.

2. Preference structures and classes of nondominated solutions

2.1. *Preference as a binary relation*

Let us consider possible outcomes of the decision *one pair at a time*. For any pair, say y^1 and y^2, one and only one of the following can occur:

(i) we are convinced that y^1 is better than or preferred to y^2, denoted by $y^1 > y^2$;

(ii) we are convinced that y^1 is worse than or less preferred to y^2 denoted by $y^1 < y^2$;

(iii) we are convinced that y^1 is equivalent or equally preferred to y^2, denoted by $y^1 \sim y^2$; or

(iv) we have no sufficient evidence to say either (i), (ii) or (iii), denoted by $y^1 \, ? \, y^2$, thus, the preference relation between y^1 and y^2 is *indefinite* or not yet clarified.

Note that each of the above statements involves *a comparison or relation* between a pair of outcomes. The symbols '>', '<', '~' and '?' are *operators* defining the comparison and relations. Specifying whether '>', '<', '~' or '?' is defined for each pair of Y. Any revealed preference information, accumulated or not, can then be represented by a subset of the Cartesian Product $Y \times Y$ as follows:

Definition 2.1. (i) A preference based on $>$ (respectively $<$, '$\sim$', or ?) is a subset of $Y \times Y$, denoted by $\{>\}$ (respectively $\{<\}$, $\{\sim\}$, or $\{?\}$), so that whenever $(y^1, y^2) \in \{>\}$ (respectively $\{<\}$, $\{\sim\}$, or $\{?\}$) $y^1 > y^2$ (respectively $y^1 < y^2$, $y^1 \sim y^2$, or $y^1 \, ? \, y^2$).

(ii) For convenience, we also define $\{\gtrsim\} = \{>\} \cup \{\sim\}$; $\{>?\} = \{>\} \cup \{?\}$; $\{\gtrsim ?\} = \{\gtrsim\} \cup \{?\}$, etc.

(iii) By a preference (or revealed preference) structure we mean the collection of all the above preferences. As such structures are uniquely determined by $\{>\}$, $\{\sim\}$, $\{?\}$) (see Remark 2.1(iii)), a preference structure will be denoted by $\mathscr{P}(\{>\}, \{\sim\}, \{?\})$ or simply by $\mathscr{P}$.

Remark 2.1. (i) Note that $>$, $<$, $\sim$ and ? are operators, while $\{>\}$, $\{<\}$, $\{\sim\}$ and $\{?\}$ are sets of revealed preference information.

(ii) The sets $\{>\}$ and $\{<\}$ are *symmetric*. That is, $(y^1, y^2) \in \{>\}$ iff $(y^2, y^1) \in \{<\}$. Thus, if $\{>\}$ is known, then $\{<\}$ is also known and vice versa.

(iii) $\{>\}$, $\{<\}$, $\{\sim\}$ and $\{?\}$ form a partition of $Y \times Y$. In view of (ii),