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Reminiscences related to graph theory

Sometimes it is useful to return to the past. In this short contribution, I intend to describe how I became interested in graph theory.

It began as follows. In around 1951, a prominent Slovak mathematician Štefan Schwarz (1914–1996) asked me, then a 25year-old postdoc., officially to read and evaluate Professor Anton Kotzig's doctoral (DrSc.) dissertation on graph theory. He said: "It is quite simple, there is just one book on the topic, D. Königs's Theorie der endlichen und unendlichen Graphen; you will find there everything that is known on the subject." Although I was at that time interested only in linear algebra, Euclidean geometry and classical algebraic geometry, I accepted the task. Ever since, I have used the graph-theoretical view whenever I could.

Soon afterwards my official duties in the Institute of Mathematics of the Czechoslovak Academy of Sciences, where I became employed, were linked with numerical mathematics (in fact, numerical algebra). In this connection, I became interested in the relationship of the combinatorial structure of zero–nonzero entries in the (square) matrix of a system of linear equations with the structure of matrices arising after one or more steps of the usual Gaussian elimination. In fact, independently of the seminal Seymour V. Parter's paper [12] (our access to the literature was then very limited) I found the complete solution (elimination graphs) [2].

I was also interested in the combinatorial structure of inverse matrices. First in the special case (which generalizes the acyclic case of triangular matrices) when the directed graph of the matrix is e-simple, i.e. it has the property that each arc (directed edge) is contained in, at most, one cycle [3,4]. I showed that there always exists an elimination ordering for which all elimination graphs are e-simple as well. I also used bigraphs (directed bipartite graphs) [5] for working with such structures.

As I mentioned, I was particularly interested in Euclidean n-dimensional geometry, especially simplex geometry. I was then able to formulate necessary and sufficient conditions for the distribution of acute, right and obtuse dihedral interior angles in such n-dimensional simplex. The result can be easily visualized as follows. There is always one edge opposite each interior angle. Color this edge red, white or blue, according to whether the opposite angle is acute, right or obtuse, respectively. Then the coloring is completely characterized by the fact that the set of red edges connects the set of all $n+1$ vertices of the simplex.

Of course, it follows that every n-simplex has at least n acute dihedral interior angles, and that there exist simplices with exactly n interior angles acute (the red edges have to form a tree) and all $\binom{n}{2}$ remaining angles right. I called such simplices [6] right simplices. They have simple properties, in various directions generalizing properties of a right triangle. For instance, the red edges (cathetes) are mutually perpendicular. A particularly simple right simplex is such that the red edges form a path (I called them later Schläfli simplices since Schläfli (1814–1895) used them, even in the non-Euclidean case, for measuring volumes; he called them Orthoscheme). The center of the circumscribed hypersphere is then in the middle of the longest edge. Schläfli simplices are also characterized by the fact that all their two-dimensional faces are right triangles.

I cannot resist mentioning an amusing relationship between so-called hyperacute simplices (i.e. simplices having no obtuse interior angles) and connected resistive electrical networks (they contain only connecting branches with resistors). I found in about 1962 that there is a one-to-one relationship between such a network (with n outlets) and (the congruence class of) a hyperacute $(n-1)$-simplex [7]. I was very disappointed to learn much later that D.J.H. Moore published this fact in his dissertation in 1968. More about applications of graphs and matrices in Euclidean geometry can be found in chapter 66 of the Handbook of Linear Algebra [8].

I should also mention my result [9] in which I proved that tridiagonal matrices and their simultanously permutation similar matrices are the only square matrices whose rank cannot be diminished by two from the maximal by changing the entries on the main diagonal. This means in the graph-theoretical setting that Colin de Verdiére's invariant is equal to one only in this case.

In around 1970, quite a few years ago, I thought about the problem of how the geometric shape of a sufficiently nice planar region could be approximated by the underlying square grid graph, namely the graph obtained by drawing a region on a quadruled paper, the vertices of which would be the grid points in the interior of the region and the edges, those which connect the neighbors; in particular, which graph invariants could be preserved if we move or rotate the paper

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or if we change the size of the grid. Also, what happens if we similarly use a paper with a regular triangular grid. I came to the conclusion that in the continuous (and not discrete as above) case the behaviour of the free membrane with the shape of the region should play an important role. This is reflected by the eigenvalue problem for the vibration of the membrane with the Neumann boundary condition. In the discrete case, the corresponding tool in the first approximation could be the Laplacian of the grid graph.

This lead me to the study of the properties of the Laplacian matrix of a general graph and the definition of algebraic connectivity [10]. The confirmation of usefulness of the theory was the fact [11] that for an arbitrary connected (even positively edge-weighted) graph, the subgraph induced by vertices in which the eigenvector of the Laplacian corresponding to the algebraic connectivity has positive coordinates, is again connected.

I want to conclude these remembrances with the information that a detailed survey about algebraic connectivity has just appeared in [1].

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